Mathematical Models of Quartz Sensor Stability

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Mathematical Models of Quartz Sensor Stability

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Abstract:

Quartz Sensor stability was mathematically modeled using data from Paroscientific pressure sensors and Quartz Seismic Sensors accelerometers. Qualitatively, we looked for models that relate to physical reality and quantitatively we looked for the best fits with the fewest free parameters and the best predictive behavior. Stability data were fit with various models and the residuals between the data and each fit were compared. Fits were derived from early data points and extrapolated to see which fits could best predict future behavior in the hope that long-term stability can be predicted based on short-term pre-deployment testing. All of the fits used worked well and the absolute drift can be modeled down by almost two orders of magnitude. In general, the “Power + Log” fit worked best and produced small residuals when extrapolated for an extended period.

Background:

Paroscientific and Quartz Seismic Sensors have developed new sensors and measurement techniques for dual-purpose Disaster Warning Systems and Geodetic Research. Earthquake and tsunami warning systems require high-resolution, high-speed, high-range sensors to measure events occurring from a fraction of a second to many hours. The slow strain build-up leading to earthquakes and tsunamis require stable measurements for decades.

The initial goal is to make geodetic measurements of uplift or subsidence (depth changes or tilt) to better than 1 cm/year at a depth of 4000 meters and a span of 1 km. At zero load, typical drift of all types of quartz crystal sensors is in the direction of higher frequency (higher output) with a magnitude of about 100 parts-per-million of full-scale per year (ppm/year) for the first year. At full-scale load, drift is in the direction of lower frequency and lower output. In both cases, initial drift tapers off rapidly in the first few months. Our goal is to model and predict the sensor stability. If the full-scale range of the pressure sensors is 4000 meters, then the requirement on pressure sensor stability is a few ppm/year. If the full-scale range of the accelerometers in our Triaxial Accelerometer Assembly is 3 G's, then the requirement on accelerometer stability is also a few ppm/year. In-situ calibration techniques have been developed to correct for the drift of quartz pressure sensors and triaxial accelerometers (Reference 1). Periodic measurements are made to a single point reference and the offset drift is subtracted from the in-situ readings. This calibration method can distinguish sensor drift from real seafloor movements. The mathematical models can aid in drift analyses and provide insights into the root causes of sensor drift.

Root Causes of Drift:

There are at least 2 causes of drift that are related to whether the sensors are unloaded or loaded. When the sensors are mostly unloaded the resonator frequencies and indicated signal outputs at both zero and full-scale increase with time. One mechanism that can cause increasing outputs is quartz crystal aging or “outgassing” whereby the resonator mass decreases and frequencies increase with time. These frequency changes must be converted to equivalent error forces through the conformance (linearization) equation. Appendix I to this report describes the relationship between offset changes and span changes. Creep is in the direction of lower outputs and is due to attachment or mechanism “creep” deflections that work against the spring rate of the mechanism to generate viscoelastic error forces. The drift effects due to outgassing and creep may be subtracted from the measured readings to correct for drift (Reference 1).

Figure 1 illustrates the combined drift effects of outgassing and creep on a 100 MPa full-scale range pressure sensor. The fits to 7 years of typical drift data at 0 were extrapolated and subtracted from 4 months of drift data held mostly at pressure A = 100 MPa. The resulting curves illustrate Drift @ 0 (outgassing), Drift @ A (creep), and Drift @ A combined.

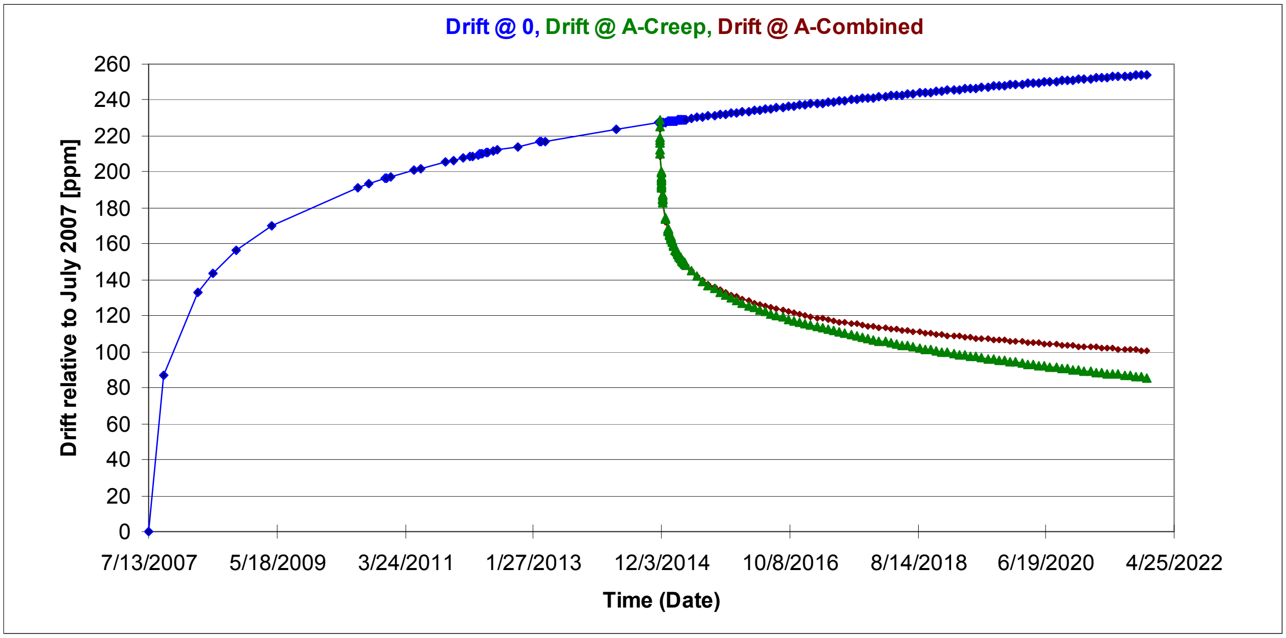


Figure 1

As shown in Figure 2, the combined drift can look quite different depending on the pressure profile.

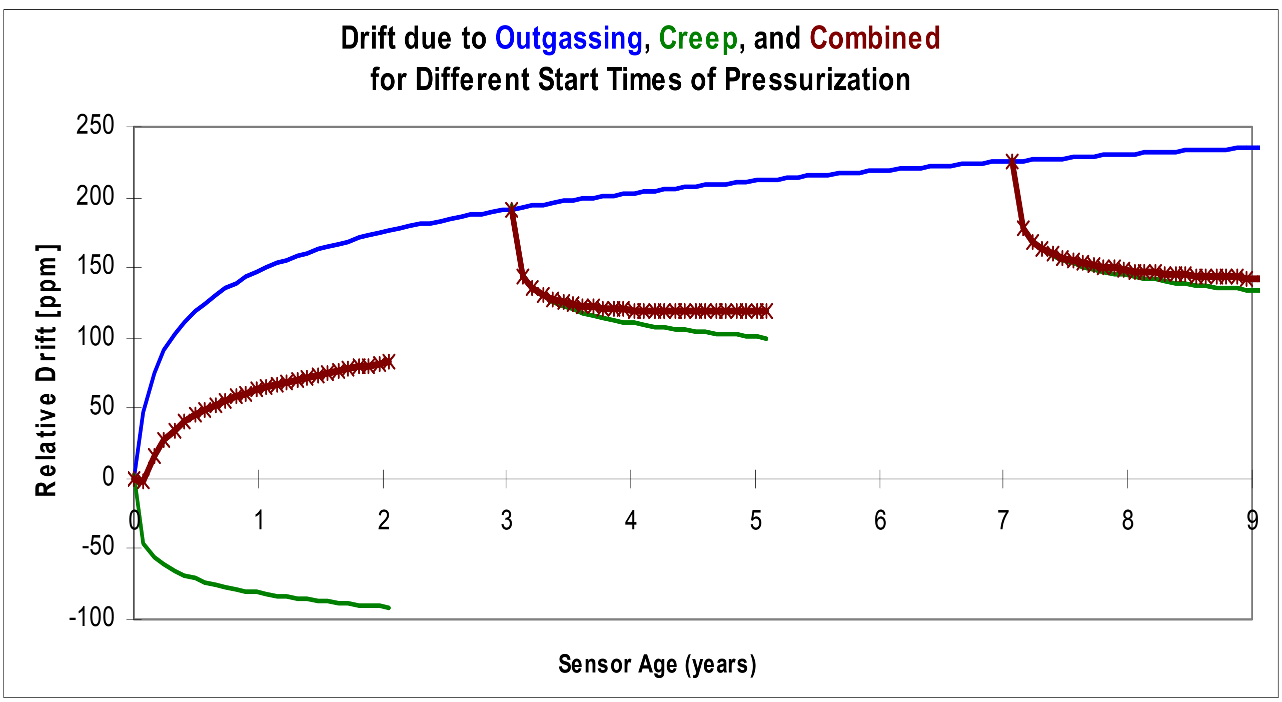


Figure 2

Mathematical Models:

Different mathematical models of quartz sensor stability have been developed including *Wearn, R. B. Jr.*, and *N. G. Larson* (*1982*), *Measurements of the sensitivities and drift of Digiquartz pressure sensors*, *Deep Sea Res.*, ***29***, *111–134*, doi:10.1016/0198-0149(82)90064-4 and Polster, et al (2009) *Effective resolution and drift of Paroscientific pressure sensors derived from long-term seafloor measurements* *http://onlinelibrary.wiley.com/doi/10.1029/2009GC002532/abstract* . Three fits were studied:

"Power + Log": P = A*t^B + C*LN(t) + D
"Log": P = A*LN(t-t0) + D
"Exp + Linear": P = A*exp(-t/t0) + C*(t/365) + D

P is the absolute pressure, t is the # of days passed and A, B, C, t0 and D are free parameters. The Drift is the difference between P and the offset D.

For **"Power + Log"** fit,
Drift = At^B + C*LN(t) and
Slope = AB*t^(B-1) + C/t

We use this equation with a condition that B < 1, so the power of t of the slope will always be negative. This means that as t goes to infinity, the slope approaches zero.

For **"Log"** fit,
Drift = A*LN(t-t_0) and
Slope = A/(t-t_0)

As t goes to infinity, the denominator approaches infinity so the slope approaches zero.

For **"Exp + Linear"** fit,
Drift = A*exp(-t/t_0) + C*(t/365) and
Slope = - (A/t_0)exp(-t/t_0) + C/365

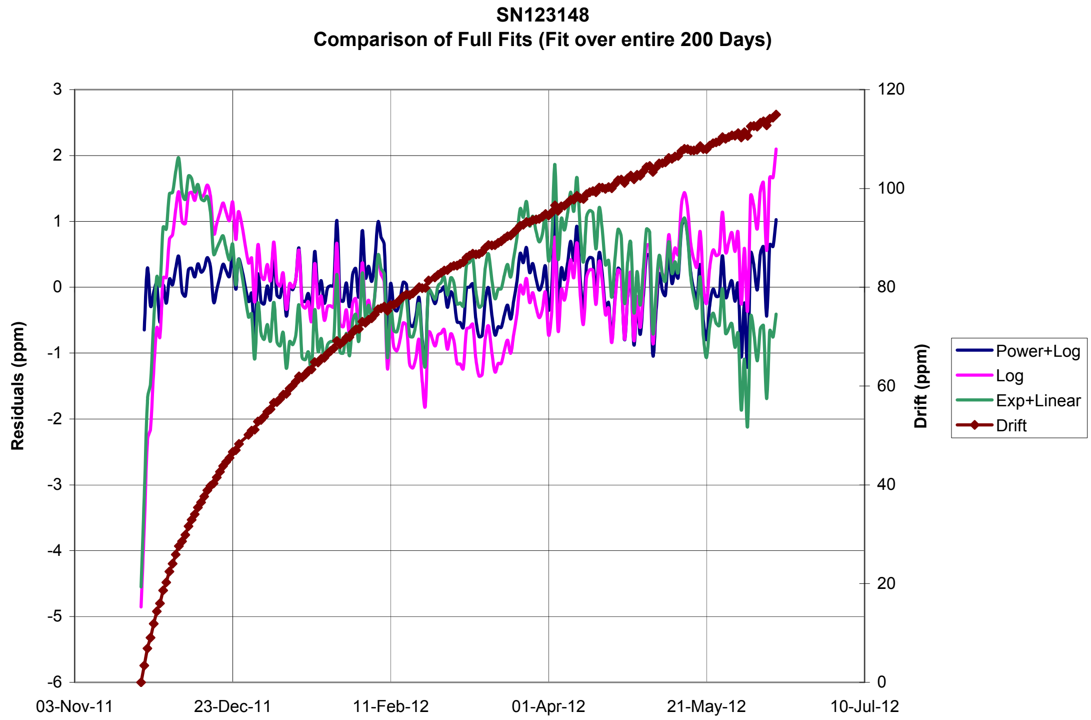
As t goes to infinity, the exponential term goes to zero (because it is an exponential decay with a negative exponent), so we are left with the slope approaching C/365 in units of drift per day.

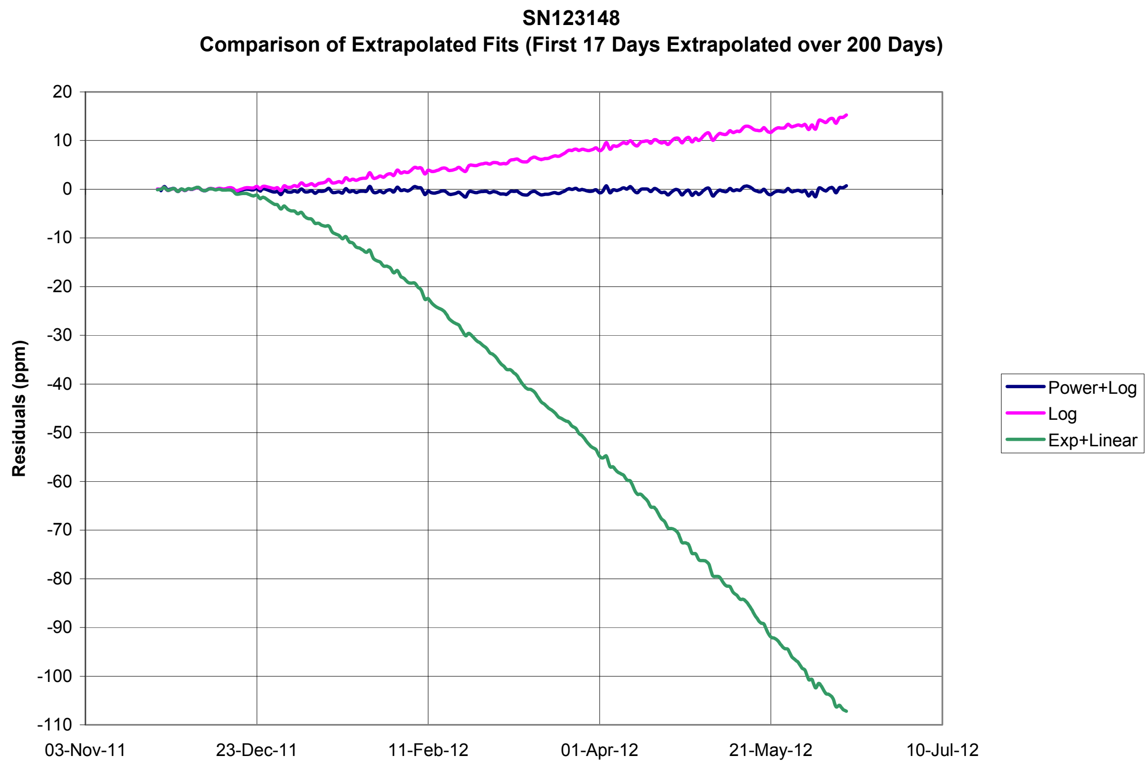
After a long period of time, the slopes of the "Log" and "Power + Log" fits approach zero, but the slopes of the "Exp + Linear" fits approach a constant value.

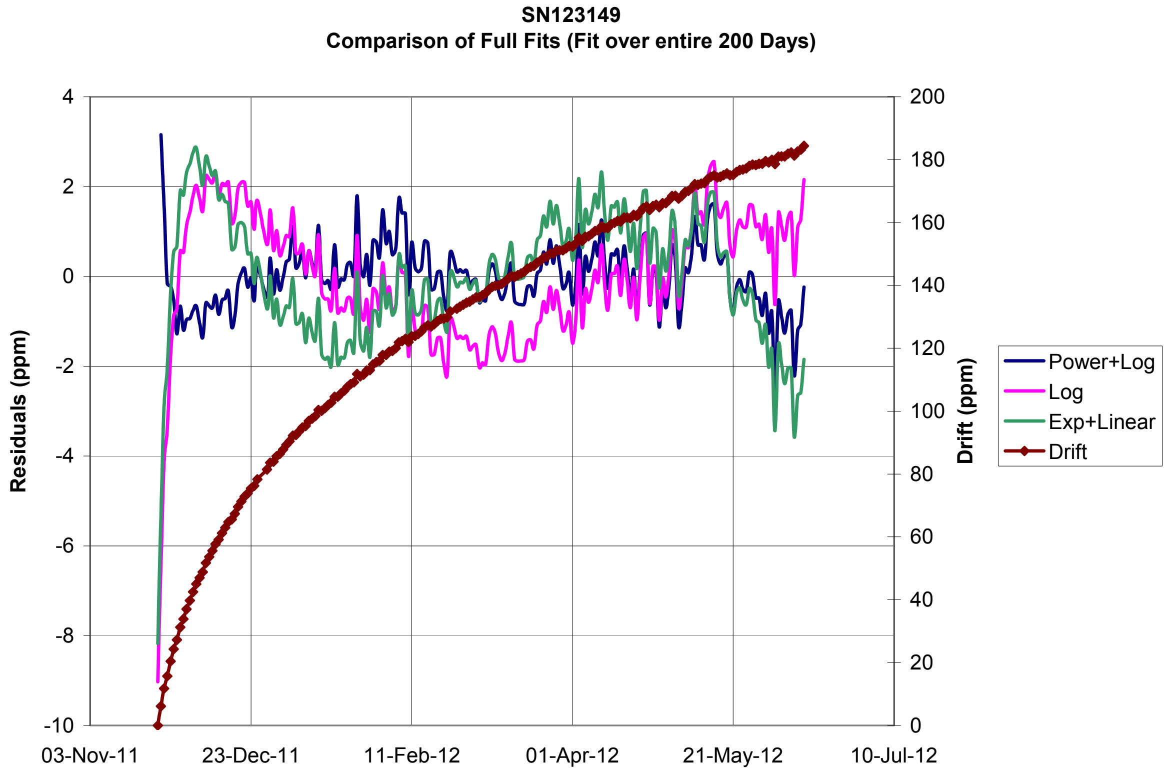
Fits to Outgassing Drift of Pressure Sensors:

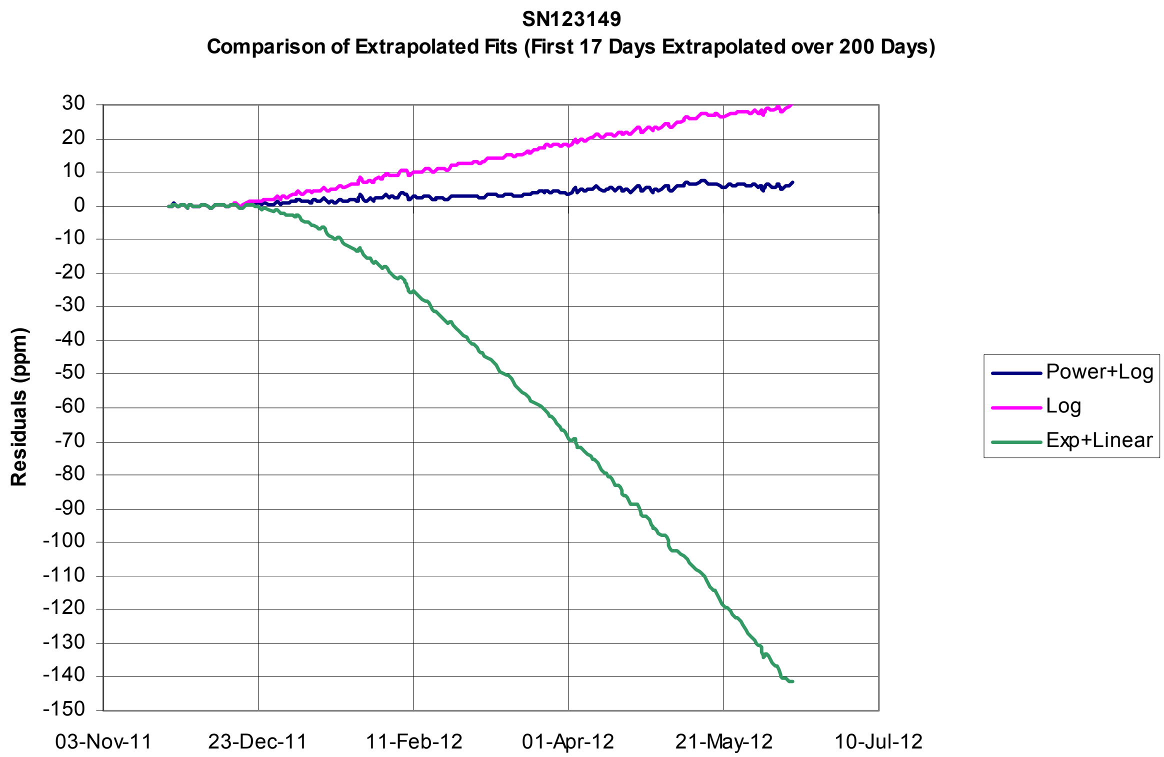
The three fits were applied to model the stability of 5 pressure sensors with 200 days of data in two ways: First the residuals between the data and the fits were calculated using all 200 days of data. Then fits were made using just the data points from the first 17 days and extrapolated to 200 days. Drift residuals are expressed in parts-per-million of full-scale (ppm). Throughout this write-up, t is the time in days and t_0, A, B, C, etc. are free parameters and their values are determined by using the Solver function in Microsoft Excel such that residuals between the selected data points and the actual fit are minimized. However, even with the same functions, same data, and same constraints, different initial values used by Solver produce different residuals. For functions involving the natural log function, the point at t = 0 was not used (since natural log of 0 is undefined).

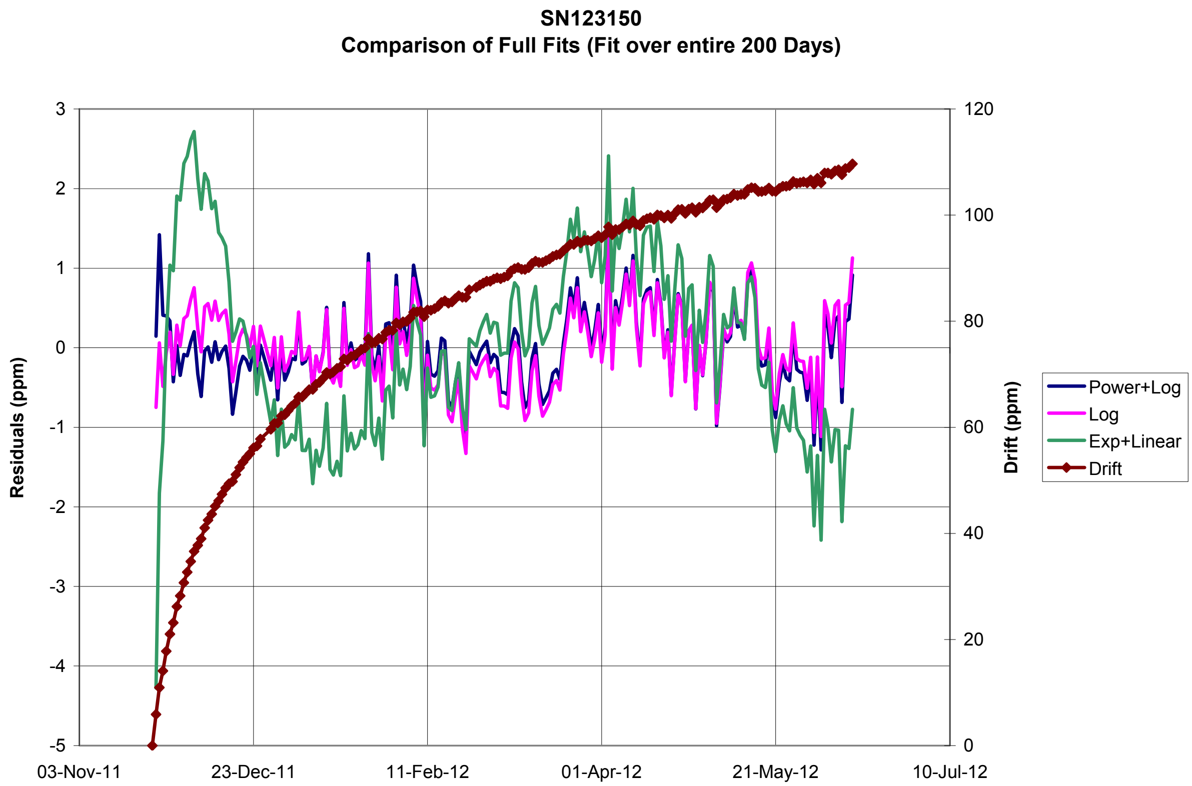
Residuals are shown on the left axis and total drift shown on the right axis (ppm of full-scale).

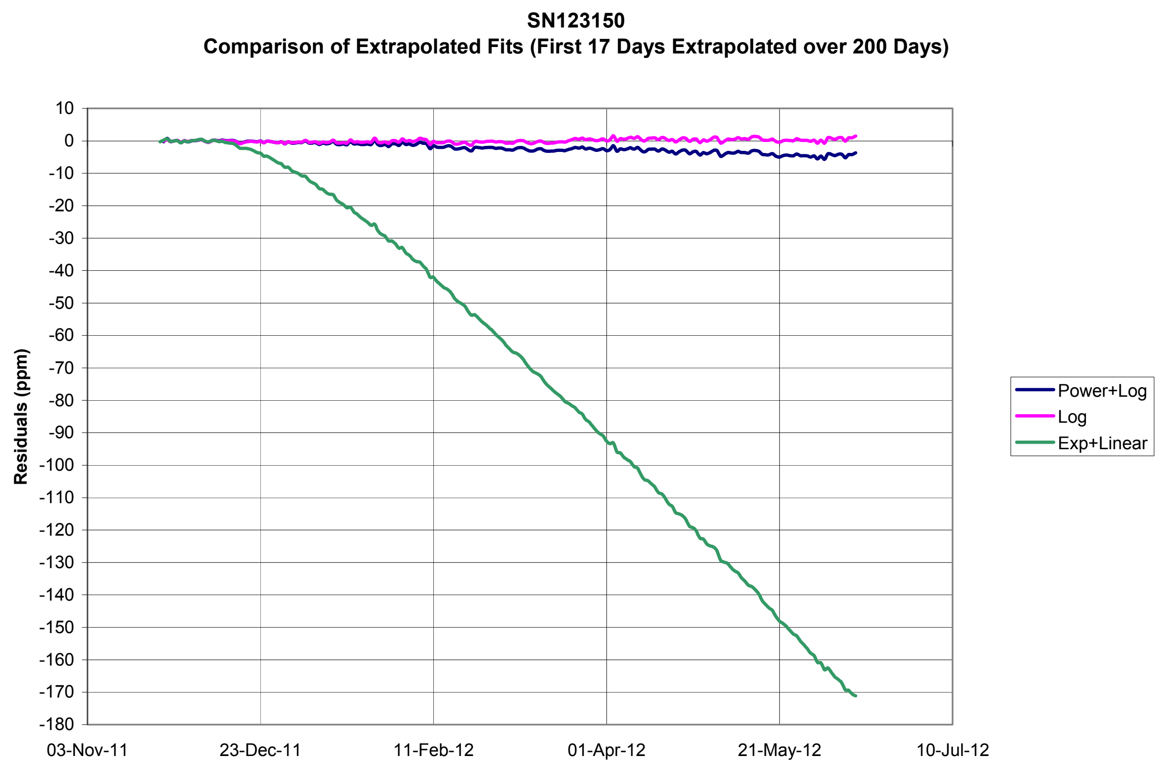




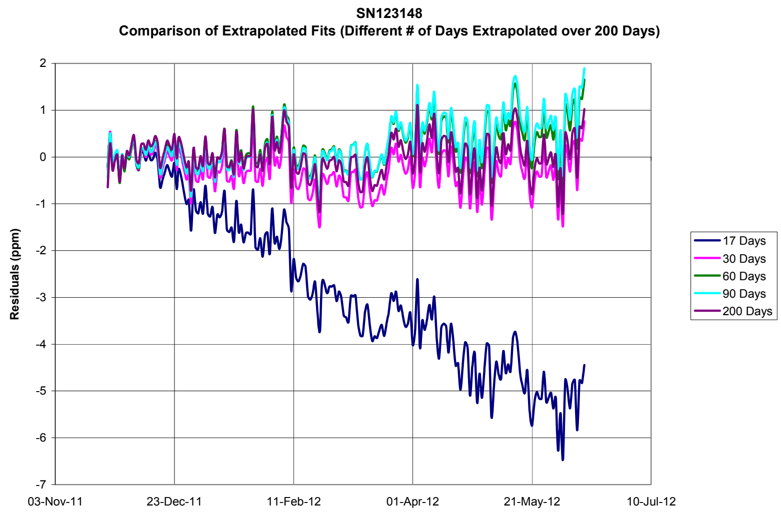

SN123149
Comparison of Full Fits (Fit over entire 200 Days)
Residuals (ppm)
Drift (ppm)
4
2
0
-2
-4
-6
-8
-10
200
180
160
140
120
100
80
60
40
20
0
03-Nov-11
23-Dec-11
11-Feb-12
01-Apr-12
21-May-12
10-Jul-12
Power+Log
Log
Exp+Linear
Drift

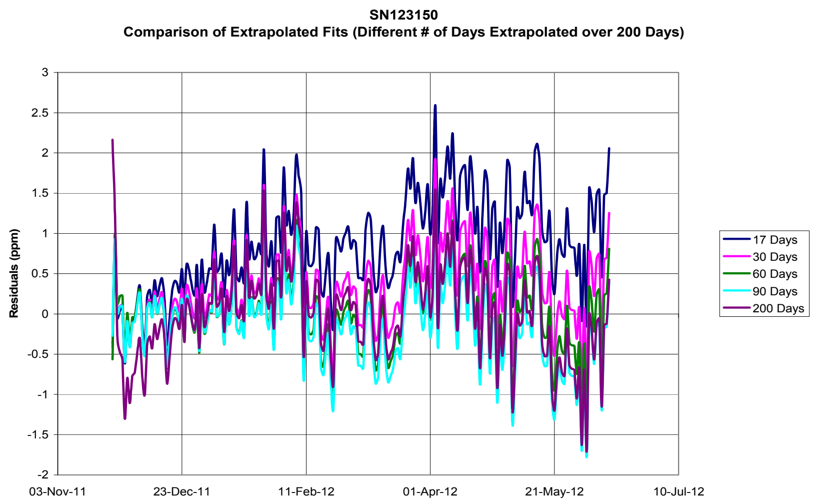

SN123149
Comparison of Extrapolated Fits (First 17 Days Extrapolated over 200 Days)
Residuals (ppm)
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
03-Nov-11
23-Dec-11
11-Feb-12
01-Apr-12
21-May-12
10-Jul-12
Power+Log
Log
Exp+Linear

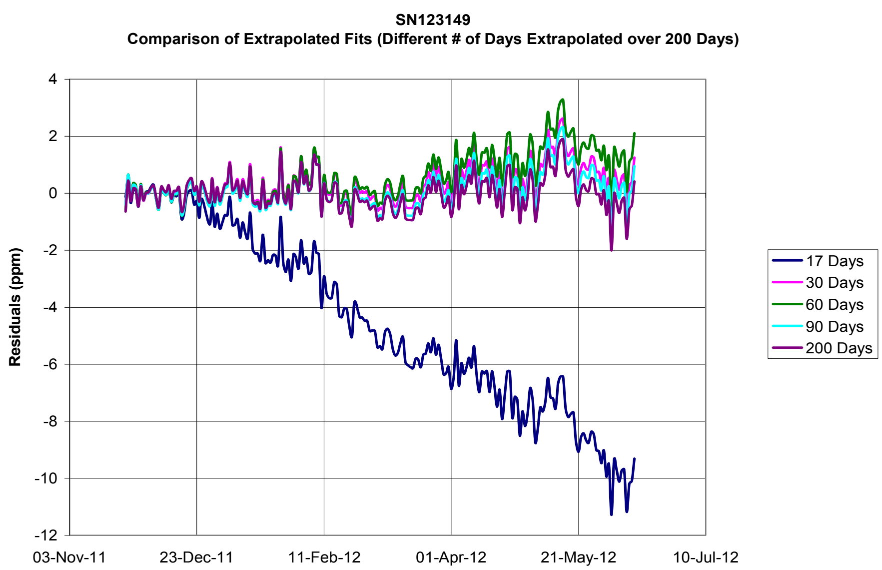

SN123150
Comparison of Full Fits (Fit over entire 200 Days)
Residuals (ppm)
Drift (ppm)
3
2
1
0
-1
-2
-3
-4
-5
120
100
80
60
40
20
0
03-Nov-11
23-Dec-11
11-Feb-12
01-Apr-12
21-May-12
10-Jul-12
Power+Log
Log
Exp+Linear
Drift


SN123150
Comparison of Extrapolated Fits (First 17 Days Extrapolated over 200 Days)
Residuals (ppm)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
03-Nov-11
23-Dec-11
11-Feb-12
01-Apr-12
21-May-12
10-Jul-12
Power+Log
Log
Exp+Linear

The Graphs below compare the residuals of the "Power + Log" fit for different baselines of 17, 30, 60, and 90 days extrapolated to the full 200 days data set.







Fits to Creep Drift of Pressure Sensors:

The curves for sensor drift due to creep are similar for both pressure sensors and accelerometers. The load-generating mechanisms for these sensors are completely different. The pressure sensors convert pressures to forces on the quartz resonators using Bourdon tubes or bellows. The seismic instruments generate forces by accelerations acting on suspended inertial masses. The attachments of the quartz resonators to the force-producing structures are similar so drifts due to creep are likely related to the attachment joints. Loads applied to the attachment joints produce viscoelastic creep (deflections) that act against the spring rates of the mechanisms to generate error forces. The quartz resonator cannot distinguish the error forces due to creep from the sensed input forces.

The ongoing, unidirectional drift due to outgassing must be subtracted from the total combined drift in order to determine the drift due to creep as shown in Figure 1. The pressure sensor creep was then modeled with the same three fits that were used to model outgassing. The Power + Log and Log fits were almost identical. This suggests that the natural log is dominant in characterizing creep. Figure 3 shows residuals with a standard deviation of 0.5 ppm.

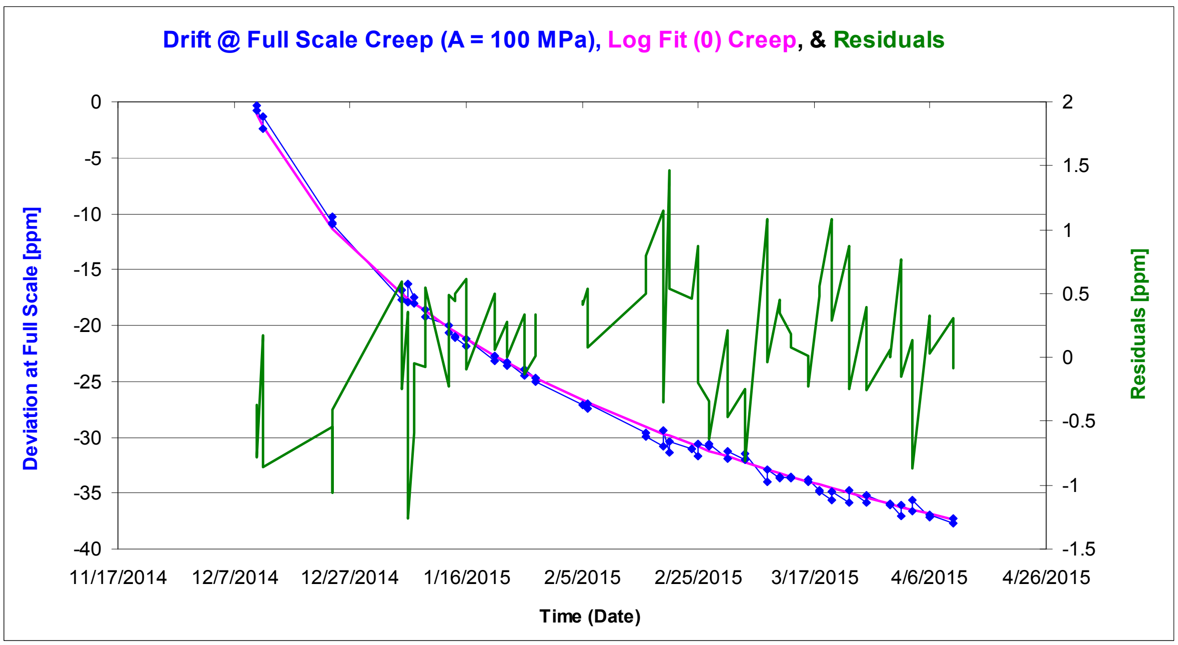
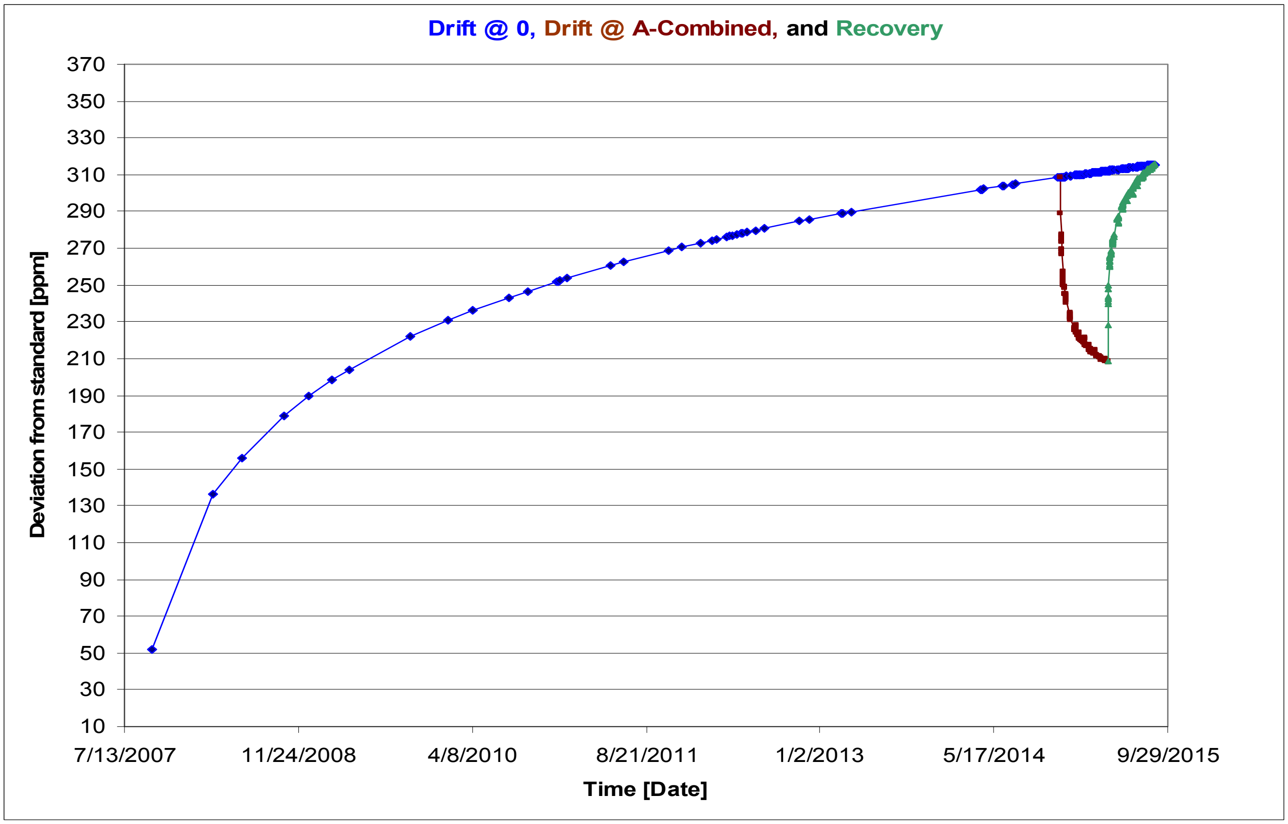
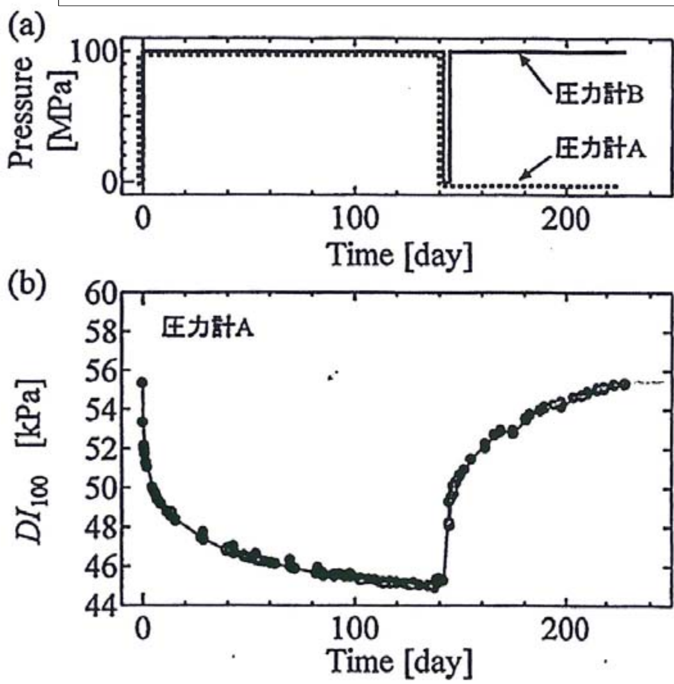


Figure 3

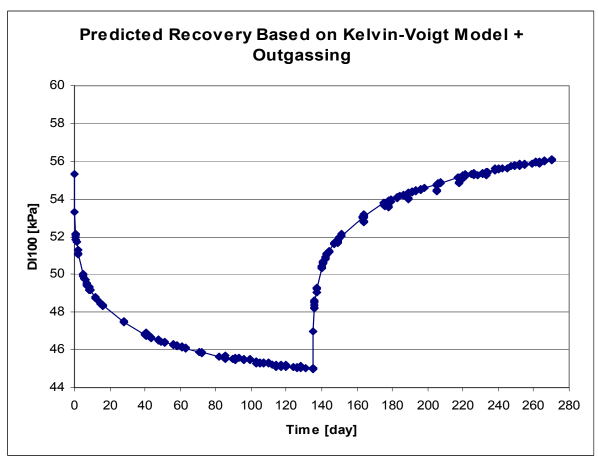
The Kelvin-Voigt viscoelastic model (Appendix II) predicts that drift due to creep is proportional to the reactive spring rate of the mechanism and the applied load. Creep is inversely proportional to the modulus. The time dependence is an exponential function with a time constant equal to the modulus divided by the viscosity. When the load is removed, a viscoelastic "recovery" occurs.

Dr. Hiroaki Kajikawa and his colleagues at the National Metrology Institute of Japan--(AIST) performed seven years of testing with quartz pressure sensors mostly at zero pressure (Reference 2). Then four months at full-scale pressure at 100 MPa was applied to measure creep, and then returned to zero pressure to monitor the viscoelastic recovery (Reference 3). The fit to the creep model was reversed in sign to model the recovery and the outgassing was added to predict the combined recovery curve. As shown in Figure 4, the predicted recovery compares well to the data plots excerpted from Reference 3.





Actual Creep & Recovery



Predicted Creep & Recovery

Figure 4

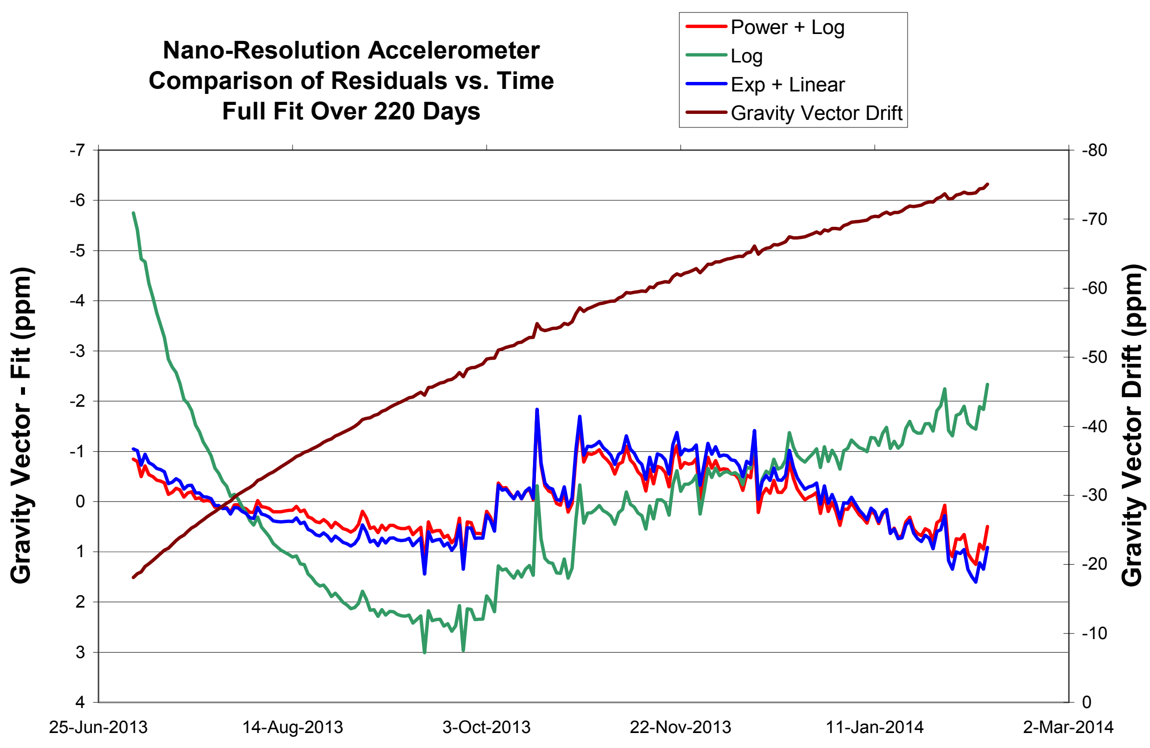
Comparison Fits on Accelerometers:

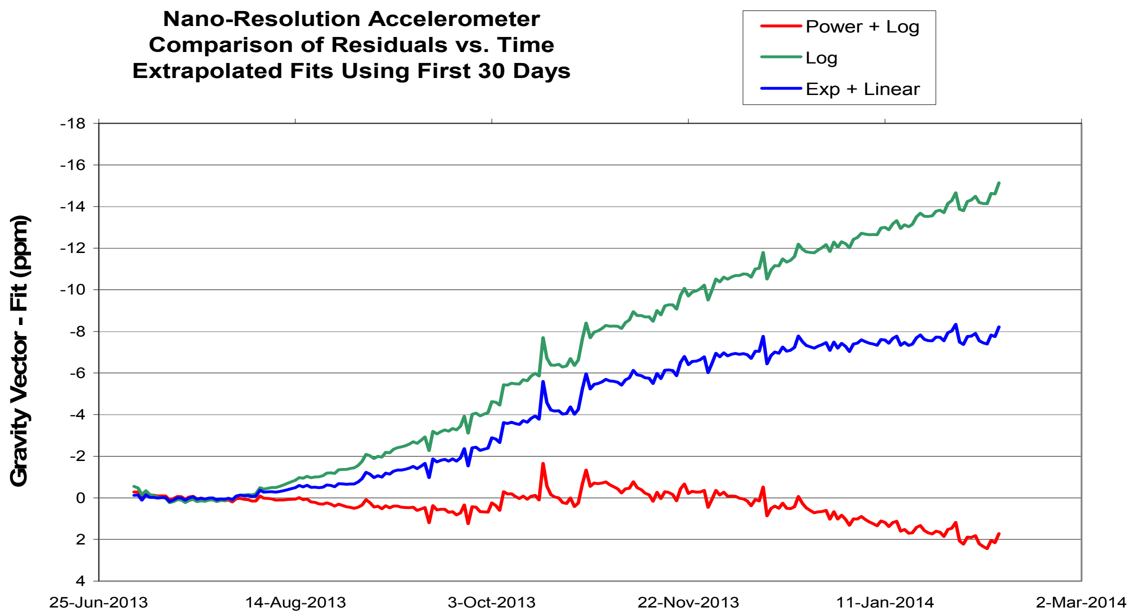
There is a need for a device and in-situ calibration method for improved seismic and geodetic measurements. Traditional strong motion sensors do not have the sensitivity or stability to make good long-term geodetic measurements. Traditional broadband seismometers and tiltmeters operate over a small fraction of Earth's 1G gravity vector and do not have the range to measure strong seismic events and have no absolute reference for long-term measurements.

The goal is to make improved surface, subsurface and submarine measurements of seismic events and geodetic measurements of earth movements such as tilt, subsidence and uplift. The initial geodesy requirement is to measure earth movements to better than 1 centimeter per year at a span of 1 kilometer. This is equivalent to a tilt of 10 micro-radians or a 10 micro-G's tilt (sine) component of Earth's 1 G static gravity.

A new Acceleration Ocean Bottom Seismometer (AOBS) has been developed and tested by Y. Fukao, H. Sugioka, A. Ito, and H. Shiobara. The AOBS contains a Triaxial Accelerometer that is calibrated with an internal alignment matrix such that measurements of Earth's gravity vector are rotationally invariant with respect to the direction of Earth's plumb line. Drift of the Triaxial Accelerometer is compensated for by measuring the changes in the measured values of the Earth's invariant static gravity vector.

The 3 fits were used to model the stability of a Triaxial Accelerometer using the invariant G vector as a reference. The first graph below shows small residuals using the full 220 days data set. However, the "Power + Log Fit" is much better than the "Exp + Linear Fit" when only the first 30 days are used to fit the entire 220 days of data as shown in the second plot below. The accelerometer drifts in the same direction as the pressure sensors; however, the scales have been inverted because of the sign convention used for positive G. The "Power + Log Fit" is good to a few ppm of the 2G full-scale accelerometers.





Conclusion:

Mathematical models of stability were evaluated to determine the best fits that can distinguish Quartz Sensor drift from real seafloor movements. These models can be applied to pre-deployment calibration data, in-situ re-calibration test data, and post-deployment stability measurements. Stability data were fit with various models and the residuals between the data and each fit were compared. Fits were also derived from early data points and extrapolated to see which fits could best predict future behavior.

Both of the presumed root causes of drift, (outgassing and creep) were successfully modeled to a few parts-per-million and a standard deviation less than 1 ppm.

Because of the common quartz crystal "core" technology, these mathematical models may be applied to the pressure sensors made by Paroscientific, Inc. and the accelerometers and tiltmeters made by Quartz Seismic Sensors, Inc.

Acknowledgements:

We thank H. Kajikawa, H. Iizumi, and M. Kojima of the National Metrology Institute of Japan for their excellent work in developing the 0-A-0 calibration method and for supplying data and analyses used in this paper.

We would also like to acknowledge the excellent work of Y. Fukao, H. Sugioka, A. Ito, and H. Shiobara in their development of a new Acceleration Ocean Bottom Seismometer (AOBS) that uses a triaxial assembly of nano-resolution quartz crystal accelerometers.

References:

1. J.M. Paros and T. Kobayashi, "Calibration Methods to Eliminate Sensor Drift", G8097, Paroscientific, Inc., Technical Note
2. H. Kajikawa and T. Kobata, "Reproducibility of calibration results by 0-A-0 pressurization procedures for hydraulic pressure transducers", Measurement Science and Technology, 25, (2014).
3. H. Kajikawa and T. Kobata. "Long-term drift of hydraulic pressure transducers constantly subjected to high pressure." Proceedings of the 32nd Sensing Forum, Osaka, Japan, 10-11 September 2015. pp.261-266.

Appendix I - Relationship between Changes in Offset and Span

The equation that characterizes the outputs of Quartz Resonator Sensors under load is:

$P = C[(1 - \tau_0^2/\tau^2) - D(1 - \tau_0^2/\tau^2)^2]$

Where:

τ is the period output,

τ_0 is the period output at P = 0,

C is the scale factor; and

D is the linearization factor.

To first order, $P = C[1-\tau_0^2/\tau^2]$

The nominal change in output period under full scale load is ±10 %. The change in period with the resonator under compression is nominally +10 % and under tension –10 %.

Let $\tau_{fs} = a\tau_0$ where a = 1.1 (for compression) or 0.9 (for tension)

The pressure at zero is: $P_0\ (\tau = \tau_0) = 0$

And the pressure at full scale is: $P_{fs}\ (\tau = \tau_{fs}) = C[1 - \tau_0^2/(a\tau_0)^2]$

$P_{fs}\ (\tau = \tau_{fs}) = C[1 - (1/a^2)]$

The span = $P_{fs} - P_0$ $\quad$ $\text{Span} = P_{fs}\ (\tau = \tau_{fs}) - P_0\ (\tau = \tau_0) = C[1 - (1/a^2)]$

For a very small change (drift), δ, in the period output, then the new pressure output, P', is:

$$P' = C[1-\tau_0^2/(\tau+ \delta)^2]$$

The new pressure outputs at 0 and full scale are:

$$P_0'(\tau = \tau_0) = C[1 - \tau_0^2/(\tau_0 + \delta)^2]$$

$$P_{fs}'(\tau = \tau_{fs}) = C[1 - \tau_0^2/(a\tau_0 + \delta)^2]$$

The change in offset due to drift δ is the new pressure output at zero, P_0'.

$$\Delta\text{Offset} = P_0'(\tau = \tau_0) - P_0 = C[1 - \tau_0^2/(\tau_0 + \delta)^2] -0$$

$$\Delta\text{Offset} = C[(2\tau_0\delta + \delta^2)/(\tau_0^2 + 2\tau_0\delta + \delta^2)]$$

Since $\tau_0 >> \delta$, $\quad$ $\Delta\text{Offset} = C(2\delta/\tau_0)$

The span = $P_{fs} – P_0$, and any change in the span due to a drift, (change in period, δ,), is the difference between the new span after the drift is induced and the old span without drift:

$$\Delta Span = (New\ Span – Old\ Span)$$

$$\Delta Span = (P_{fs}' – P_0') – (P_{fs} – P_0)$$

$$= C[1 – \tau_0^2/(a\tau_0 + \delta)^2] – C[1- \tau_0^2/(\tau_0 + \delta)^2] – C[1 – (1/a^2)]$$

$$= C[-\tau_0^2/(a^2\tau_0^2 + 2a\tau_0\delta + \delta^2) + \tau_0^2/(\tau_0^2 + 2\tau_0 \delta + \delta^2) – 1 + 1/a^2$$

$$\Delta Span = C[– 1/(a^2 + 2a(\delta/\tau_0)) + 1/(1 + 2(\delta/\tau_0)) – 1 + 1/a^2]$$

$$\Delta Offset = C(2\delta/\tau_0)$$

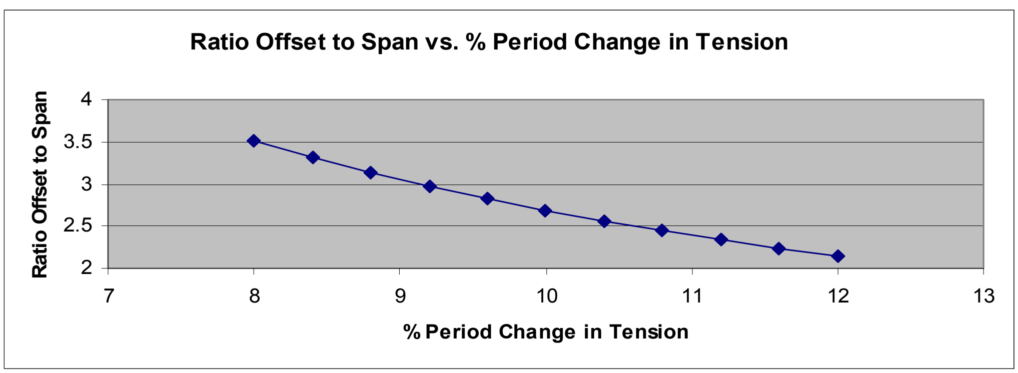
$$\Delta Offset / \Delta Span = [a^3 + 2a^3(\delta/\tau_0) + 2a^2(\delta/\tau_0) + 4a^2(\delta/\tau_0)^2] / [1-a^3]$$

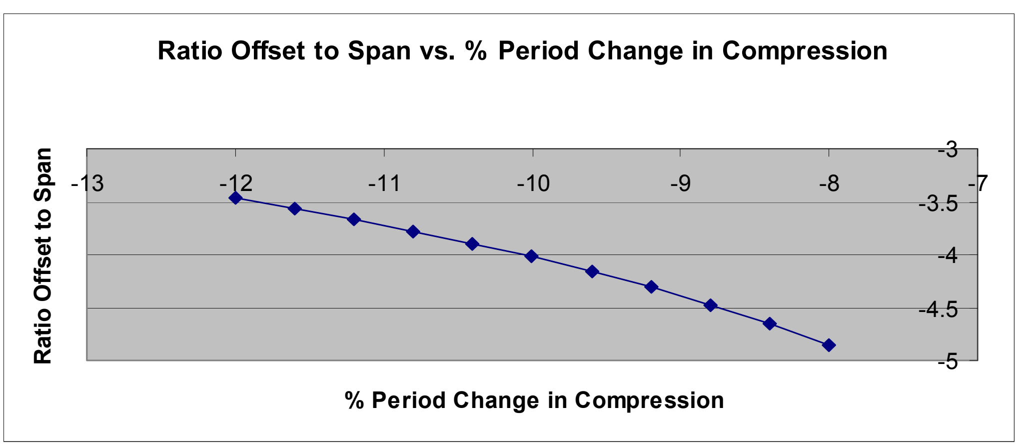
Since (δ/τ_0) is very small, the numerator can be approximated with the leading term a^3 and:

$$\mathbf{\Delta Offset / \Delta Span = a^3/(1 – a^3)}$$

Examples:
For a = 1.1 (compression), $\Delta Offset / \Delta Span = 1.1^3/(1 – 1.1^3) = – 4.02$
For a = 0.9 (tension), $\Delta Offset / \Delta Span = 0.9^3/(1 – 0.9^3) = +2.69$

Plotted below are the values of ΔOffset / ΔSpan for various “a” values expressed as percent period change in Tension and Compression.





Appendix II Kelvin–Voigt Model of Creep Drift

The Kelvin–Voigt model represents loads applied to a purely viscous damper, D, and purely elastic spring, S, connected in parallel as shown in Figure 1.

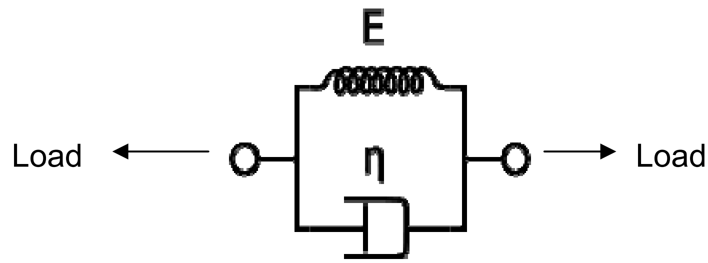


Figure 1 Kelvin–Voigt Model

Since the spring and damper are arranged in parallel, the strains in each component are equal:

Strain = $\varepsilon_{Total} = \varepsilon_D = \varepsilon_S$

The total stress will be the sum of the stress in each component:

Stress = $\sigma_{Total} = \sigma_D + \sigma_S$

The stress in the spring equals the modulus of elasticity, E, times the strain in the spring. The stress in the damper equals the viscosity, η, times the rate of change of strain in the damper. Thus in a Kelvin–Voigt material, stress σ, strain ε, and their rates of change with respect to time *t* are given by:

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt}$$

The above equation may be used for both shear stress and/or normal stress load applications.

Response to a Stress Step Function

If a constant stress, σ_0, is suddenly applied to a Kelvin–Voigt material, then the strain approaches the strain of the pure elastic material, $\frac{\sigma_0}{E}$, as a decaying exponential:

Strain $(t) = \frac{\sigma_0}{E}(1 - e^{-\lambda t})$ **,** where *t* is time and λ is the relaxation rate, $\lambda = \frac{E}{\eta}$

If the stress is suddenly removed at time, t_1, then the deformation is retarded in the return to zero deformation:

Strain $(t > t_1) = \varepsilon(t_1)(1 - e^{-\lambda(t-t_1)})$

Figure 2 shows the deformation versus time when constant stress on the material is applied suddenly at time, t = 0, and suddenly released at the later time, t_1.

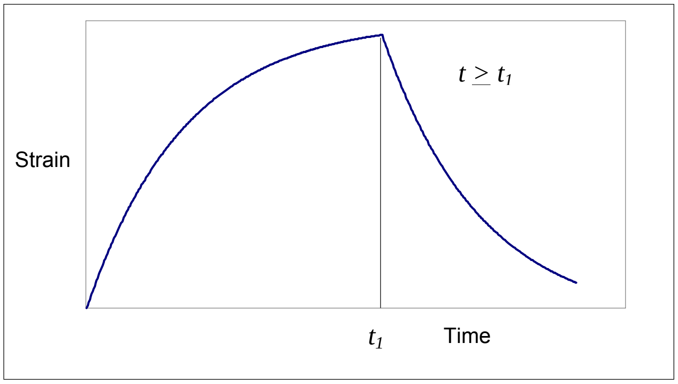


Figure 2: Deformation versus time for sudden application and release of constant stress

The deformation, ΔL, over the length of the attachment, L, is:

$$\Delta L = \left(\frac{L\sigma_0}{E}\right)(1 - e^{-\lambda t})$$

The reactive spring rate of the mechanism, K, acts against the deformation, ΔL, to generate a creep force, ΔF.

$$\Delta F = \left(\frac{KL\sigma_0}{E}\right)(1 - e^{-\lambda t})$$

Drift due to the creep force, ΔF, can be expressed as a fraction of the full-scale force, F_{FS}:

$$\frac{\Delta F}{F_{FS}} = \left(\frac{KL\sigma_0}{F_{FS}E}\right)(1 - e^{-\lambda t})$$

The drift due to creep is proportional to the reactive spring rate of the mechanism and the applied load. Creep is inversely proportional to the modulus. The time dependence is an exponential function with a time constant equal to the modulus divided by the viscosity.

References:

Meyers, Marc A., and Krishan Kumar Chawla. "Creep and Superplasticity." *Mechanical Behavior of Materials*. 2nd ed. Cambridge: Cambridge UP, 2009. 653-688. Print.