The importance of ADCP alignment with GPS in moving-boat streamflow measurements

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ABSTRACT

This paper addresses the importance of the alignment of an acoustic Doppler current profiler (ADCP) with a global positioning system (GPS) in moving-boat streamflow measurements. It presents a mathematical analysis of the discharge bias induced by a misalignment angle. A small misalignment angle may cause a significant bias in a transect discharge. The bias consists of non-directional and directional components. The directional bias is proportional to the ratio between boat velocity and water velocity. In a normal condition of ADCP streamflow measurements, however, the directional bias in transect discharges can be approximately cancelled in the average discharge of reciprocal transects, even if heading-dependent errors are involved in the misalignment. This paper also presents a trial-and-error method for estimating the misalignment angle. We analyzed the transect discharge data obtained from a field measurement on the Yangtze River at the Huangling Temple hydrology station located about 5 km downstream from the Three Gorges Dam to gain insights on the effect of misalignment. Results of this case study suggest that the data for transect discharges must be processed to remove directional bias prior to the Type A evaluation of the random uncertainty of the measured discharge. Otherwise, the estimated Type A uncertainty would be false and misleading.

1. Introduction

Acoustic Doppler current profilers (ADCPs) have been widely used by hydrologists all around the world for streamflow measurements in recent years. An ADCP is commonly mounted on a survey boat or a small float to transverse a river when making streamflow measurements. This measurement procedure is known as moving-boat ADCP method. Detailed discussion on the ADCP technology and moving-boat ADCP method can be found in, e.g. Simpson [1] Oberg et al. [2]; and Mueller et al. [3].

During a transect from the start bank to the end bank, an ADCP measures the water velocity relative to the ADCP. This relative velocity is known as water tracking velocity; it is also known as apparent water velocity [4]. The actual water velocity can be obtained by subtracting the boat velocity from the relative velocity. There are two commonly used methods for deriving boat velocity: bottom tracking and a global positioning system (GPS). In the bottom tracking method, an ADCP transmits a special acoustic pulse to measure the bottom velocity relative to the ADCP, known as bottom tracking velocity. In the case that no moving-bed exists, boat velocity is the negative vector of the bottom tracking velocity. An advantage of the bottom tracking method is that, because both water tracking and bottom tracking velocities are measured within the same coordinate system (i.e. ADCP coordinates), discharge calculation is independent of ADCP internal compass error or the rotation of the ADCP coordinates. However, in a moving-bed condition, the bottom tracking method may induce a negative bias in water velocity and discharge measurement. The moving-bed error is attributed to the movement of sediment on or near the streambed [5]. The U. S. Geological Survey (USGS) policy requires that a moving-bed test be made prior to making any discharge measurements [2]. The moving-bed induced discharge bias may be addressed by the loop-method [5], stationary ADCP method (e.g. Refs. [3,6]), or by using a GPS (e.g. Refs. [7,8]).

The use of a GPS in the moving-boat ADCP method has increased in recent years because: (1) the accuracy of GPS has been increasing and the cost has been decreasing, (2) an integrated ADCP-GPS system can effectively resolve the moving-bed problem, particularly for large rivers, and (3) geo-referenced velocity data are required for mapping a velocity field in water bodies.

The boat velocity derived from GPS data is under the earth coordinate system (N, E). The ADCP data for relative velocity are collected under the ADCP coordinate system (X, Y). An ADCP uses the heading data provided by an internal magnetic compass, GPS compass, or external gyrocompass to transfer the components of relative velocity vectors collected in the (X, Y) coordinate frame into those in the (N, E) coordinate frame. If the relative angle between the two coordinate
frames is exactly known and heading data are free from errors, after the transformation, the direction of a relative velocity vector will be in the true direction. In practice, however, this ideal scenario may not exist. This is because the transformed (N', E') coordinate frame would not exactly align with the (N, E) coordinate frame: one of the frames may have rotated relative to the other by an unknown angle, known as misalignment angle. Thus, after the transformation, the direction of a relative velocity vector may not be in the true direction. According to Osinski [9]; the misalignment angle (denoted by α) is “the difference between the real angle φ and the information about angle φ received by ADCP.”

When heading data are provided by an external gyrocompass, the misalignment angle can be expressed as the sum of the mounting angle error (denoted by Δmounting) and heading error (denoted by Δheading) [9].

\[ α = Δ_{\text{mounting}} + Δ_{\text{heading}} \]  

(1)

The mounting angle is the angle between the ADCP coordinate frame and the external gyrocompass coordinate frame. The mounting angle error Δmounting is a physical misalignment. On a boat, no matter how much effort is made, an ADCP may not be mounted with its coordinate frame exactly aligned with the gyrocompass coordinate frame, or with knowing the exact mounting angle. That is, a small mounting angle error Δmounting is unavoidable.

When heading data are provided by an internal magnetic compass, the local magnetic variation (MagVar, denoted by ΔMagVar) is an additional contribution to the misalignment angle. That is

\[ α = Δ_{\text{mounting}} + Δ_{\text{MagVar}} + Δ_{\text{heading}} \]  

(2)

Since an internal magnetic compass is usually precisely installed on an ADCP electronics board and aligned with the ADCP coordinate frame, the mounting angle error Δmounting, if any, might be much smaller than the other two components and can be ignored.

In general, the heading error of a gyrocompass or a magnetic compass depends on heading (i.e. the direction of boat course); it is usually not a constant (e.g. Refs. [4,9,10]). Thus, the misalignment angle may not be a constant.

It is well known that, a small misalignment angle may cause a significant error in velocity measurements (e.g. Refs. [4,9,11]). It is also well known from the experiences of ADCP users including the author that a small misalignment angle may cause a significant error in discharge measurements. The misalignment induced error is a bias (or systematic) error according to the conventional definition of bias error. For simplicity, we call it ‘bias’ throughout this paper. Moreover, the bias in transect discharges is directional and often has a great difference between reciprocal transects, which could cause a significant bias in the Type A (random) uncertainty estimated with the statistical analysis of transect discharge data.

A number of studies focused on the velocity bias induced by the ADCP misalignment with respect to the (N, E) coordinate frame and on the methods for estimating the misalignment angle (e.g. Ref. [9–13]). Mueller [4] assessed the heading error induced biases in ADCP velocity and discharge measurements. However, to the author’s knowledge, no study has addressed the bias in Type A uncertainty analysis caused by the ADCP misalignment with respect to a GPS.

This paper addresses the importance of the ADCP alignment with a GPS in moving-boat streamflow measurements. In the following, section 2 presents a mathematical formulation for the bias in transect discharges, induced by a misalignment angle. Section 3 presents formulas for estimating the bias in the average discharge of reciprocal transects. Section 4 presents a trial-and-error method for estimating the misalignment angle. Section 5 presents a case study of moving-boat ADCP streamflow measurements on the Yangtze River to gain insights on the effect of misalignment. Section 6 presents conclusion.

2. Bias in a transect discharge

Let \( \vec{u} \) denote the water velocity vector, \( \vec{u}_t \) the relative velocity vector (derived from ADCP’s water tracking), and \( \vec{u}_b \) the boat velocity vector. For the discussion in this paper, we assume that \( \vec{u}_b \), measured by an ADCP, and \( \vec{u}_b \), measured by a GPS, are free of random and bias errors. Fig. 1 shows a vector diagram when an ADCP is well aligned with a GPS, i.e. the misalignment angle \( α = 0 \). For the convenience of our discussion, we assume that the direction of the streamflow is to the east. In Fig. 1, \( \theta \) is the direction of the boat velocity and \( \omega \) is the direction of the relative velocity, measured from the north. The relationship between these velocity vectors is written as [4].

\[ \vec{u} = \vec{u}_t + \vec{u}_b \]  

(3)

If \( \vec{u}_b = 0 \), i.e. the boat is stationary, \( \vec{u} = \vec{u}_t \). On the other hand, if \( \vec{u}_b \neq 0 \), i.e. the boat is travelling in the same direction of the streamflow and at the same velocity as the water velocity, \( \vec{u} = \vec{u}_b \).

The discharge (denoted by \( Q \)) measured from a transect is known as the transect discharge. We assume that edge discharges are ignored. The transect discharge \( Q \) can be calculated with a flux integral

\[ Q = \int_{0}^{T} \int_{0}^{H} \vec{u} \cdot \hat{k} \ dz \ dt \]  

(4)

where \( T \) is the transect time from the start point to the end point, \( z \) is the upwards coordinate measured from the river bottom \( (z = 0) \), \( z = H \) is the water surface, \( \vec{u} \times \hat{k} \) is the unit vector normal to the boat path, \( \hat{k} \) is the unit vector in the vertical direction, \( dz \) is the differential depth, and \( dt \) is the differential time. Details on discharge calculation using Eq. (4) can be found in, e.g. RDI [14] and TRDI [15].

Substituting \( \vec{u} = \vec{u}_t + \vec{u}_b \) into Eq. (4) yields

\[ Q = \int_{0}^{T} \int_{0}^{H} (\vec{u}_t \times \vec{u}_b) \cdot \hat{k} \ dz \ dt \]  

(5)

where \( f = (\vec{u}_t \times \vec{u}_b) \cdot \hat{k} \), which is the magnitude of the cross-product of the relative velocity and boat velocity vectors. It is interesting to compare Eq. (5) with Eq. (4) and notice that the discharge \( Q \) can also be calculated directly using the relative velocity vector. Referring to the vector diagram shown in Fig. 1, \( f \) can be calculated as

\[ f = |\vec{u}_t| |\vec{u}_b| \sin(\theta - \omega) \]  

(6)
We assume that \( Q \) is the “true” discharge and \( f \) is the “true” magnitude of the cross-product. Now consider that an ADCP has a misalignment angle \( \alpha \) with respect to a GPS. In this situation, \( \mathbf{u}_B \) has no change because \( \mathbf{u}_B \) is derived from GPS data. Thus, both the magnitude and direction of \( \mathbf{u}_B \) (i.e. \( |\mathbf{u}_B| \) and \( \theta \)) are the same as those shown in Fig. 1. Let \( \mathbf{u}_B' \) and \( \mathbf{u}_B \) denote the new relative velocity and water velocity vectors respectively. The magnitude of \( \mathbf{u}_B' \) has no change because the magnitude of water tracking velocity is independent from the rotation of coordinates. It is the same as the magnitude of \( \mathbf{u}_B \) (i.e. \( |\mathbf{u}_B| \)), but the direction of \( \mathbf{u}_B' \) rotates by the misalignment angle \( \alpha \) (refer to Fig. 2). However, both the direction and magnitude of \( \mathbf{u}_B' \) change and are different from those of \( \mathbf{u}_B \) due to the misalignment angle \( \alpha \) (compare Fig. 2 with Fig. 1).

Let \( f' = (\mathbf{u}_B' \times \mathbf{u}_B) \cdot \mathbf{k} \) refer to the vector diagram shown in Fig. 2. \( f' \) can be written as

\[
\alpha \sin(\theta - (\omega + \alpha))
\]

(7)

Let \( Q' \) denote the transect discharge affected by the misalignment angle \( \alpha \). \( Q' \) can be written as

\[
Q' = \frac{\int_0^T H f' dz}{dt}
\]

(8)

Thus, the bias in a transect discharge, denoted by \( \delta Q \), is written as

\[
\delta Q = Q' - Q = \frac{\int_0^T H (f' - f) \ dz}{dt}
\]

(9)

It is well known that the moving-boat ADCP method does not require a straight-line path perpendicular to the streamflow. In principle, a survey boat can make an arbitrary path and the measured discharge will be the same if an ADCP is well aligned with a GPS or boat velocity is derived from bottom tracking. This feature is known as the path-independence principle [16]. Nevertheless, a straight-line or approximate straight-line path perpendicular to the streamflow is typically preferred because (1) it results in the most appropriate representation of the channel cross-sectional area, and (2) it will minimize heading changes that may cause additional errors.

Consider a special case where a boat transverses the stream along a path that is normal to the streamflow. Assume that the boat transverses to the south and the water flows to the east. Thus, \( \mathbf{u}_B \) is in the direction of \( \theta = 180^\circ \) \( \mathbf{u}_B \) is in the direction of \( 90^\circ \). Because \( \mathbf{u}_B \) is perpendicular to \( \mathbf{u}_B \), we have

\[
|\mathbf{u}_B'| \cos \omega = |\mathbf{u}_B|
\]

(10)

Substitute \( \theta = 180^\circ \) and Eq. (10) into Eq. (9). After arrangement, Eq. (9) becomes

\[
\delta Q = (\cos(\alpha - 1)) \ Q + (\sin(\alpha)) \ Q_B
\]

(11)

where \( Q_B = \int_0^T H \int_0^H dz \ dt \).

Notice from Eq. (11) that, the bias in a transect discharge has two components: \( (\cos(\alpha - 1)) \ Q \) and \( (\sin(\alpha)) \ Q_B \). The first component is proportional to the “true” discharge \( Q \) by the factor \( \cos(\alpha - 1) \); it is negligible if the misalignment angle \( \alpha \) is small. The second component \( (\sin(\alpha)) \ Q_B \) is proportional to boat velocity; it is usually much larger than the first component. It is important to note that the second component \( (\sin(\alpha)) \ Q_B \) is proportional to water velocity, for which the sign of \( \alpha \) is either positive or negative, depending on the boat path with respect to the streamflow. If it is positive in a direction of a transect, it will be negative in the reverse direction of the reciprocal transect, regardless of the sign of \( \alpha \).

The relative bias (RB) in a transect discharge is defined as

\[
RB_{TB} = \frac{\delta Q}{Q} = \cos \alpha + \frac{Q_B}{Q} \sin(\alpha - 1)
\]

(12)

It can be readily derived that the discharge ratio \( Q_B/Q \) can be approximated using the ratio between the mean boat velocity magnitude \( V_B \) and the mean water velocity magnitude \( V_W \). Eq. (12) can be rewritten as

\[
RB_{TB} = \cos \alpha + \frac{V_B}{V_W} \sin(\alpha - 1)
\]

(13)

As indicated by Eq. (1) or (2), the misalignment angle \( \alpha \) includes a heading error \( \delta \omega \). Mueller [4], provided a comprehensive analysis of the heading error induced bias in ADCP velocity and discharge measurements. In the special case where the boat path is normal to the streamflow, his formula (32) for the relative bias in a transect discharge, induced by a heading error, is essentially identical to Eq. (13).

It should be mentioned that we used a slightly different approach to that presented in Mueller [4] for deriving the bias in transect discharges. Mueller’s approach is based on the cross-product of water velocity and boat velocity vectors. Our approach is based on the cross-product of relative velocity and boat velocity vectors, i.e. Eq. (5). Both approaches are mathematically valid and give the same results. However, our approach helps to understand and visualize the relationship between the three velocity vectors involved in the ADCP velocity and discharge measurements as shown in Figs. 1 and 2.

Eq. (13) indicates that the relative bias in a transect discharge depends on the misalignment angle \( \alpha \) and the velocity ratio \( V_B/V_W \). Fig. 3 shows the relationship between \( RB_{TB} \) and \( \alpha \) at \( V_B/V_W = 0.5, 1, \) and

![Fig. 3. Relative bias in a transect discharge as a function of the misalignment angle.](image-url)
It can be seen from Fig. 3 that a ± one-degree misalignment angle at the velocity ratio 2 would produce a relative bias of +3.47% and −3.51% respectively; a ± 5-degree misalignment angle at the velocity ratio 2 would produce a relative bias of +17.1% and −17.8% respectively. That is, the bias in a transect discharge can be very large if the misalignment angle \( \alpha \) is large and/or the velocity ratio \( V_B/V_W \) is large.

3. Bias in the average discharge of reciprocal transects

It is a common practice that, under a steady flow condition, a moving-boat ADCP streamflow measurement consists of multiple reciprocal transects (at least one reciprocal pair). Making reciprocal transects is to reduce potential directional biases [2,3], especially important when using GPS because of the much greater potential for directional biases to occur.

Consider a measurement consisting of \( m \) pairs of reciprocal transects under a steady flow condition. Let \( Q_{L-R} \) denote the transect discharge measured with the boat travelling from the left bank to the right bank and \( Q_{R-L} \) from the right bank to the left bank. We assume that the boat path is normal to the streamflow and the boat keeps a constant speed during the measurement. The average discharge of the \( m \) pairs of reciprocal transects is taken as the measured discharge. That is

\[
Q_{\text{average}} = \frac{1}{2m} \sum_{i=1}^{m} (Q_{L-R}^{i} + Q_{R-L}^{i}).
\]  

Eq. (14) is a simple and practical discharge estimator that does not require the knowledge of the misalignment angle \( \alpha \). The validity and effectiveness of this discharge estimator has been demonstrated with many field measurements. A case study is presented later in this paper.

In the following, we analyze the bias in \( Q_{\text{average}} \) caused by the ADCP misalignment with respect to a GPS.

Eq. (13) indicates that the bias in a transect discharge has two components: cosine and sine. The cosine-component is non-directional. That is, if \( \alpha \) is positive in a direction of a transect, it will be negative in the reverse direction of the reciprocal transect, regardless of the sign of \( \alpha \).

In general, the misalignment angle \( \alpha \) is not a constant. In this study, we considered magmatic compass that is commonly built into an ADCP. According to the two-cycle (hard and soft iron) error model for magmatic compass (e.g. Ref. [17]), \( \alpha \) can be expressed as the sum of three components

\[
\alpha = \alpha_0 + \alpha_f + \alpha_d
\]  

where \( \alpha_0 \) is a constant error that may consist of the physical misalignment, \( \alpha_f \) and \( \alpha_d \) are the one-cycle and two-cycle heading-dependent errors respectively.

\[
\alpha_f = B \sin \phi + C \cos \phi
\]  

where \( \phi \) is the compass heading, the coefficient \( B \) accounts for the fore-aft permanent magnetic field, and \( C \) accounts for the port-starboard permanent magnetic field.

\[
\alpha_d = D \sin(2\phi) + E \cos(2\phi)
\]  

where the coefficient \( D \) accounts for symmetrical arrangements of horizontal soft iron, and \( E \) accounts for asymmetrical arrangements of horizontal soft iron.

Let \( RB_{Q(R-L)} \) denote the RB for a transect discharge measured with the boat travelling from the left bank to the right bank and \( RB_{Q(L-R)} \) from the right bank to the left bank. We assume that

\[
RB_{Q(R-L)} = \cos \alpha_{L-R} - \frac{V_B}{V_W} \sin \alpha_{L-R} - 1
\]  

Then, for the reciprocal transect, \( RB_{R-L} \) should be

\[
RB_{Q(R-L)} = \cos \alpha_{R-L} - \frac{V_B}{V_W} \sin \alpha_{R-L} - 1
\]  

Eq. (18) and (19) are essentially identical to formulas (36) and (37) in Mueller [4].

For the left-to-right transect, substituting \( \alpha_{L-R} = \alpha_0 + \alpha_f + \alpha_d \) into Eq. (18) yields

\[
RB_{Q(L-R)} = \cos(\alpha_0 + \alpha_f + \alpha_d) + \frac{V_B}{V_W} \sin(\alpha_0 + \alpha_f + \alpha_d) - 1
\]  

For the right-to-left transect, the heading changes 180°. The one-cycle error \( \alpha_f \) changes the sign because \( \sin(180 + \phi) = -\sin(\phi) \) and \( \cos(180 + \phi) = -\cos(\phi) \) but the two-cycle error \( \alpha_d \) is the same because \( \sin(360 + 2\phi) = \sin(2\phi) \) and \( \cos(360 + 2\phi) = \cos(2\phi) \). Thus, \( \alpha_{R-L} = \alpha_0 - \alpha_f + \alpha_d \). Eq. (19) becomes

\[
RB_{Q(R-L)} = \cos(\alpha_0 - \alpha_f + \alpha_d) - \frac{V_B}{V_W} \sin(\alpha_0 - \alpha_f + \alpha_d) - 1
\]  

Using the sum and difference formulas in trigonometry, Eq. (20) can be rewritten as

\[
RB_{Q(L-R)} = \cos(\alpha_0 + \alpha_f + \alpha_d) + \frac{V_B}{V_W} \sin(\alpha_0 + \alpha_f + \alpha_d) - 1
\]  

Eq. (21) can be rewritten as

\[
RB_{Q(R-L)} = \cos(\alpha_0 + \alpha_f + \alpha_d) - \frac{V_B}{V_W} \sin(\alpha_0 + \alpha_f + \alpha_d) - 1
\]  

Averaging Eqs. (22) and (23) yields the relative bias in the average discharge of reciprocal transects

\[
RB_{Q(\text{average})} = \cos(\alpha_0 + \alpha_f + \alpha_d) - \frac{V_B}{V_W} \cos(\alpha_0 + \alpha_f + \alpha_d) \sin(\alpha_f) - 1
\]  

It can be seen from Eq. (24) that, the directional bias in transect discharges induced by heading-dependent errors cannot be fully cancelled with reciprocal transects; they are only partially cancelled. It is important to note that the remaining directional bias is attributable to the one-cycle error only. If \( \alpha_f = 0 \), the second term (term II) in Eq. (24) is zero and Eq. (24) reduced to

\[
RB_{Q(\text{average})} = \cos(\alpha_0 + \alpha_d) - 1
\]  

If both \( \alpha_f = 0 \) and \( \alpha_d = 0 \), or \( \alpha_{L-R} \) and \( \alpha_{R-L} \) are constants and equal (i.e. \( \alpha_{L-R} = \alpha_{R-L} = \alpha_0 \)), Eq. (25) reduces to

\[
RB_{Q(\text{average})} = \cos(\alpha_0) - 1
\]  

Eq. (26) is identical to formula (35) of Mueller [4] for the relative bias in the average discharge of a reciprocal pair of transects, induced by a constant heading error.

It is important to note that the remaining directional bias, i.e. term II in Eq. (24), is proportional to the ratio between the boat and water velocity. In a normal condition of ADCP streamflow measurements, however, term II may be significantly smaller than term I. This can be seen from their ratio \( \frac{\text{term II}}{\text{term I}} = \frac{\sin \phi V_B}{\cos \phi V_W} \). For example, at a moderate boat to water velocity ratio \( V_B/V_W = 2 \) and a nominal one-cycle error of 2°, the ratio between term II and term I is 0.07. Thus, term II is negligible and can be dropped from Eq. (24). This leads to

\[
RB_{Q(\text{average})} = \cos(\alpha_0 + \alpha_d) - 1
\]  

Eq. (27) suggests that, in a normal condition of ADCP streamflow measurements, the directional bias in transect discharges can be approximately cancelled in the average discharge of reciprocal transects, even if heading-dependent errors are involved in the misalignment. However, the non-directional bias (term I) always exists and causes a negative bias because \( \cos(\alpha_0 + \alpha_d) \cos \alpha_d \) is always smaller than unity.
4. Estimating the misalignment angle \( \alpha \)

In practice, a small misalignment angle is often unavoidable, which may cause a significant directional bias in a transect discharge. Consequently, the Type A uncertainty of the measured discharge (i.e. the average discharge of multiple transects), estimated with the statistical analysis of the directionally biased transect discharges, would be biased and unrealistic. Therefore, in order to obtain a realistic estimate of the Type A uncertainty, it is necessary to determine the misalignment angle \( \alpha \), with which to correct the misalignment induced bias in transect discharges. Note again that, in general, the misalignment angle \( \alpha \) is not a constant and depends on heading. However, to a first-order approximation and for a specific measurement, we can assume that the misalignment angle is an approximate constant. Other methods such as the traditional ‘course method’ (e.g. Refs. [15,18]) or ‘multi-epoch method’ [10] essentially employ the same assumption.

The misalignment angle \( \alpha \) can be estimated by a trial-and-error method in the post-processing of transects data using the ADCP software, e.g. WinRiver II for Teledyne RD instruments ADCPs [15]. Consider a measurement consisting of a pair of reciprocal transects made in a steady flow condition. The original velocity data may be “contaminated” by an unknown misalignment angle \( \alpha \). As a result, there is a significant difference between \( Q_{L,R}^i \) and \( Q_{R,L}^i \), denoted by \( \Delta Q_i \). First, guess a value of \( \alpha \) (say, 5°). If heading data are provided by an external gyrocompass, enter the guessed \( \alpha \) as a heading offset into WinRiver II. If heading data are provided by an internal magnetic compass, enter the guessed \( \alpha \) as a MagVar into WinRiver II. The guessed \( \alpha \) may cause an increase or decrease of \( \Delta Q \). Then, slightly increasing or decreasing the guessed value of \( \alpha \) to make \( \Delta Q \) decreases. Repeating this procedure until the guessed \( \alpha \) makes the relative \( \Delta Q \) is very small, say, less than 1%. This trial-and-error method can also be used for a measurement consisting of multiple reciprocal transects. In this situation, the criterion for determining \( \alpha \) is to minimize the differences between the transect discharges and the average discharge of all transects. By the way, to the author's knowledge, this trial-and-error procedure is used by some ADCP users to adjust the MagVar setting in WinRiver II, but it has not been reported in the ADCP literature.

The proposed trial-and-error method is robust; it can be used in the case where the misalignment angle is large, reciprocal paths are not exactly the same, or boat velocity is not uniform. Another advantage of this method is that it is applicable in a moving-bed condition because it does not rely on bottom tracking. However, it is not applicable in an unsteady flow condition. Other methods such as the traditional ‘course method’ (e.g. Refs. [15,18]) or ‘multi-epoch method’ [10] may be used to estimate the misalignment angle, regardless of steady or unsteady flow conditions because these two methods do not rely on water tracking. However, these two methods cannot be used in a moving-bed condition because both methods rely on accurate, unbiased bottom tracking.

After the misalignment angle \( \alpha \) is determined, enter it as a heading offset or MagVar into the WinRiver II software and recalculate transect discharges. The recalculated average discharge of all transects is taken as the measured discharge and the associated Type A uncertainty can then be estimated with the statistical analysis of the recalculated transect discharges.

It should be emphasized that, the trial-and-error method accounts for the misalignment angle that is assumed to be an approximate constant. As shown in section 3, the misalignment angle may consist of a constant error, a one-cycle error, and a two-cycle error. The constant and two-cycle error-caused directional bias can be cancelled, but the one-cycle error-caused directional bias can only be partially cancelled, with reciprocal transects. Therefore, it is unavoidable that an unknown bias in the average discharge may exist. Nevertheless, the trial-and-error method should provide a first-order approximation for estimating the misalignment angle.

It should be pointed out that the trial-and-error method does not apply to directional bias causes other than the ADCP misalignment with respect to a GPS. It is possible to have directional bias that does not result from ADCP misalignment. Directional bias could happen, although uncommon, even when using bottom tracking as the boat velocity reference. For example, directional bias could be caused by improper ADCP mounting or improper boat operation.

5. A case study

5.1. Measurement and data collected

In 2002, the Changjiang (i.e. the Yangtze River) Water Resources Commission (CWRC) and Teledyne RD Instruments collaborated in a project to measure the Yangtze River discharge with an ADCP integrated with multiple external sensors including a GPS, a depth sounder, and a gyrocompass. The field measurements were conducted on September 20, 21, and 22 at the Huangling Temple hydrology station located about 5 km downstream from the Three Gorges Dam. Four different ADCPs were used: a 150 kHz Workhorse ADCP, a 300 kHz Broadband ADCP, a 300 kHz Workhorse Monitor ADCP, and a 600 kHz Workhorse Rio Grande ADCP. Details on the field measurements and major results were described in Marsden et al. [19].

In this study, we analyzed the transect discharge data collected with the 300 kHz Workhorse Monitor ADCP on September 20, 2002 to gain insights on the effect of the ADCP misalignment with respect to a GPS. This measurement consisted of 6 transects in 3 reciprocal pairs. During the measurement period (between 10:00 and 11:00), the river flow was steady; the river was about 490 m wide and 50 m deep (maximum); the mean water velocity was about 0.82 m/s; the mean boat velocity was about 1.62 m/s. The ADCP was configured with the following settings: cell size = 1 m, number of cells = 60, blank distance = 2 m, single water tracking ping, and single bottom tracking ping. The heading data provided by the ADCP internal magnetic compass were used in the analysis.

Table 1 shows the measured transect discharges. The transect discharges were calculated using the WinRiver II software with the velocity data referenced to bottom tracking (BT) and GPS (GGA data were used to drive boat velocities). The relative difference between the GPS and BT referenced discharges are also shown in Table 1.

The transect discharges referenced to GPS shown in Table 1 have

<table>
<thead>
<tr>
<th>Transect</th>
<th>Data file name</th>
<th>Start bank</th>
<th>Number of ensembles</th>
<th>Duration (s)</th>
<th>Discharge referenced to BT</th>
<th>Discharge referenced to GPS</th>
<th>Relative difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DATA000</td>
<td>Left</td>
<td>341</td>
<td>340</td>
<td>11248</td>
<td>11229</td>
<td>−0.17</td>
</tr>
<tr>
<td>2</td>
<td>DATA001</td>
<td>Right</td>
<td>343</td>
<td>342</td>
<td>11617</td>
<td>11658</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>DATA002</td>
<td>Left</td>
<td>313</td>
<td>312</td>
<td>11503</td>
<td>11661</td>
<td>1.57</td>
</tr>
<tr>
<td>4</td>
<td>DATA003</td>
<td>Right</td>
<td>330</td>
<td>329</td>
<td>11513</td>
<td>11413</td>
<td>−0.87</td>
</tr>
<tr>
<td>5</td>
<td>DATA005</td>
<td>Left</td>
<td>280</td>
<td>279</td>
<td>11360</td>
<td>11657</td>
<td>2.61</td>
</tr>
<tr>
<td>6</td>
<td>DATA006</td>
<td>Right</td>
<td>311</td>
<td>310</td>
<td>11600</td>
<td>11301</td>
<td>−2.58</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11474</td>
<td>11487</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: The transect discharges referenced to GPS were calculated in the post-processing with the correction of the misalignment angle of −2° in the original data.
accounted for a misalignment angle of −2°, which was determined with the trial-and-error method described in section 4. It can be seen from Table 1 that the transect discharges referenced to BT and those referenced to GPS agree reasonably well. The relative difference between the GPS and BT referenced discharges ranges from −2.58% to 2.61% with the average 0.11%. This suggests that: (1) the estimated misalignment angle −2° is valid, and (2) moving-bed is very small and negligible at the site during the measurement. However, we do not expect that the GPS referenced discharges exactly agree with the BT referenced discharges. This 0.11% mean difference may be attributable to heading-dependent errors, errors in ADCP water and bottom tracking, and errors in GPS derived boat velocity.

5.2. Bias examination

In order to examine the discharge bias induced by the ADCP misalignment with respect to the GPS, we reprocessed the original data in the WinRiver II software with several assumed misalignment angles (i.e. constants α, which were entered into WinRiver II as MagVar values). Table 2 and Table 3 show the transect discharges calculated with the assumed negative and positive misalignment angles, respectively. We assume that, 11487 m³/s, the average discharge of all six transects without misalignment is the “true” discharge. The relative bias in the average discharge with each assumed misalignment angle is also calculated and shown in Tables 2 and 3. Fig. 4 shows the relative bias data in comparison with the RBQ(average) models: Eqs. (24), (26) and (27), as a function of α.

It can be seen from Tables 2 and 3 that the difference between each pair of Q′(average) and Q″(average), i.e. directional bias, increases dramatically with increasing the misalignment angle. However, the average discharge of the 3 pairs of reciprocal transects [i.e. the discharge estimator Eq. (14)] agree very well with the “true” discharge 11487 m³/s when the misalignment angle is small. The relative bias in Q(average), is only 0.34% and −1.38% for a misalignment angle of −2° and −5° respectively; it is less than 1% for a positive misalignment angle smaller than 10°. The relative bias in Q(average) becomes large when the misalignment angle is large. This suggest that Eq. (14) should be used only when the misalignment angle is small, say, within ± 5°.

It can be seen from Fig. 4 that the relative bias data are not symmetric around the zero misalignment. The data would be symmetric only if the misalignment angle is a constant αo, as indicated by Eq. (26). Since the data are not symmetric, according to Eq. (24) or (27), there must be a significant contribution from the two-cycle error to the misalignment angle α. Compass calibration data is not available in this dataset (the compass may not be calibrated at the site, otherwise, compass calibration data would be included in the original ADCP data file). Thus, the actual one-cycle and two-cycle errors are unknown. We applied α2 = 0.2 and α2 = −6.5 to make the RBQ(average) model, Eq. (24), fit the relative bias data (the velocity ratio Vg / Vw = 2 was used in the calculation). The value α2 = −6.5 seems large, but it is possible for an uncalibrated compass. In general, after an effective calibration, the one-cycle or two-cycle error is typically within ± 1°. Note from Fig. 4 that the prediction of Eq. (27) is very close to that of Eq. (24), suggesting that the remaining directional bias, i.e. term II in Eq. (24), is negligible for this measurement. Fig. 4 also suggests that Eq. (26) or Eq. (27) is a valid approximation when the overall misalignment angle is small (say, within ± 5°).

5.3. Type A (random) uncertainty estimation

The total uncertainty of a moving-boat ADCP streamflow measurement can be divided into two components: random uncertainty (i.e. precision limit) and systematic uncertainty (i.e. bias limit) [20]. The random uncertainty accounts for all random error sources encountered at the measurement site, including ADCP system noise in depth and velocity measurements, pitch, roll and heading variation/errors, and ambient turbulence [21]. The systematic uncertainty accounts for calibration errors and application errors. Major sources of application errors include operator errors, site conditions, assumptions made in discharge calculations, and algorithms for estimation of invalid data [20]. The discharge bias induced by the ADCP misalignment with respect to a GPS may be largely considered as an operator (i.e. application) error because it is operator’s responsibility to identify the misalignment induced directional bias and make correction for Δheading and ΔMagVar. However, Δheading is not an operator error but a limitation of the compass and magnetic environment in which the measurement is being made. Detailed discussion on the bias error sources and estimation of the bias limit can be found in Huang [20].

The random uncertainty of a moving-boat ADCP streamflow measurement can be estimated with the statistical analysis of transect discharge data collected in steady flow conditions. This is known as the
Table 4
Bias and uncertainty estimation results for measured discharges.

<table>
<thead>
<tr>
<th>Velocity reference</th>
<th>$Q_{\text{average}}(m^3/s)$</th>
<th>Relative bias (%)</th>
<th>REU (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom tracking (BT)</td>
<td>11474</td>
<td>-0.11</td>
<td>1.05</td>
</tr>
<tr>
<td>GPS without misalignment</td>
<td>11487</td>
<td>0.00</td>
<td>1.45</td>
</tr>
<tr>
<td>GPS with $-2'$ misalignment</td>
<td>11434</td>
<td>0.34</td>
<td>7.60</td>
</tr>
</tbody>
</table>

Type A evaluation of random uncertainty according to the GUM (Guide to the Expression of Uncertainty in Measurement [22]). The Type A (random) uncertainty is an important quality parameter in the uncertainty-based statistical quality control [23]. In this case study, we considered the Type A uncertainty only.

The Type A uncertainty of the measured discharge (i.e. the average discharge of multiple transects), in terms of the expanded uncertainty at the nominal coverage probability 95%, can be estimated using a mean-unbiased estimator [23], denoted by $U_{95}$

$$U_{95} = 1.96 \frac{s}{c_4 \sqrt{n}} \text{, } n \geq 2$$

where $s$ is the sample standard deviation of transect discharges; $n$ is the total number of transects ($n = 2$ m); $c_4$ is the bias-correction factor for $s$, $c_4 = \frac{\Gamma(\frac{3}{2})}{\Gamma(\frac{1}{2})}; \Gamma(\cdot)$ stands for Gamma function; $c_4 = 0.9515$ at $n = 6$.

The relative expanded uncertainty (REU) is defined as

$$\text{REU} = \frac{U_{95}}{Q_{\text{average}}}$$

It should be emphasised that $U_{95}$ [Eq. (28)] is a mean-unbiased estimator of the true uncertainty $U_{\text{true}} = 1.96 \sigma / \sqrt{n}$, where $\sigma$ is the unknown population standard deviation. $U_{95}$ was proposed to be used for uncertainty-based measurement quality control [24]. It complies with the recently proposed unified theory of measurement errors and uncertainties [25]. Therefore, $U_{95}$ is adopted in this study. In addition, the previously recommended median-unbiased uncertainty estimator [21,26] is about the same as the mean-unbiased uncertainty estimator for the sample size greater than 10, but slightly more conservative for the sample size smaller than 10.

Again, we assume that, 11487 $m^3/s$, the average discharge of all six transects without misalignment is the “true” discharge. Table 4 shows the relative bias of the measured $Q_{\text{average}}$ referenced to BT and $Q_{\text{average}}$ referenced to GPS with the misalignment angle of $-2'$ in the original data. Table 4 also shows the relative expanded uncertainty (REU) associated with the corresponding measured $Q_{\text{average}}$, estimated with Eq. (29).

Note from Table 4 that, the measured $Q_{\text{average}}$ referenced to BT has a bias of $-0.11\%$ and a REU of 1.05%. The measured $Q_{\text{average}}$ referenced to GPS without misalignment has zero bias because it is the assumed “true” discharge; it has a REU of 1.45%. This suggests that the measured $Q_{\text{average}}$ referenced to BT ($Q_{\text{average}} = 11474 m^3/s$) and that referenced to GPS ($Q_{\text{average}} = 11487 m^3/s$) without misalignment agrees very well in terms of “trueness” and precision. Either $Q_{\text{average}} = 11474 m^3/s$ or $Q_{\text{average}} = 11487 m^3/s$ is acceptable according to the uncertainty-based quality control criterion: REU $\leq 4.09\%$, the maximum permissible relative uncertainty [23]. On the other hand, the measured $Q_{\text{average}}$ referenced to GPS ($Q_{\text{average}} = 11434 m^3/s$) with the misalignment angle $-2'$ has a bias of 0.34% and a REU of 7.60%. Although the bias is very small (only 0.34%), the Type A uncertainty is large (REU = 7.60%) and overstated because of the directional biases in the transect discharges as shown in the column 3 of Table 2. The overstated uncertainty would result in a false rejection of the measured $Q_{\text{average}} = 11434 m^3/s$ because REU = 7.60% $> 4.09\%$. Therefore, we suggest that a misalignment angle be corrected prior to the Type A evaluation of random uncertainty. That is, the uncertainty analysis should be conducted using the data for transect discharges with directional biases removed.

Otherwise, the estimated Type A uncertainty would be false and misleading.

6. Conclusion

The mathematical analysis of the discharge bias induced by the ADCP misalignment with respect to a GPS and the case study of the Yangtze River discharge measurement provide insights on the discharge bias. A small misalignment angle may cause a significant bias in a transect discharge. The bias has two components: cosine and sine. The cosine-component is non-directional; the sine-component is directional and has opposite sign in reciprocal transects. In a normal condition of ADCP streamflow measurements, the directional bias in transect discharges can be approximately cancelled in reciprocal transects, even if heading-dependent errors are involved in the misalignment. Because of this, the average discharge of reciprocal transects provides an appropriate estimate of the “true” discharge when the misalignment angle is small, say, within $\pm 5\%$. However, the Type A evaluation of the random uncertainty of the measured discharge, based on the statistical analysis of directionally biased transect discharges, would be false and misleading. Therefore, we suggest that a misalignment angle be determined using the proposed trial-and-error method and corrected prior to the Type A evaluation of random uncertainty. That is, the uncertainty analysis should be conducted using the data for transect discharges with directional biases removed.

The proposed trial-and-error method for estimating the misalignment angle (assumed to be an approximate constant) is a first-order approximation. Nevertheless, it produces acceptable results in real-world applications. An advantage of the trial-and-error method is that it is applicable in a moving-bed condition because it does not rely on bottom tracking. However, it is not applicable in an unsteady flow condition. The traditional ‘course method’ or ‘multi-epoch method’ may be used to estimate the misalignment angle, regardless of steady or unsteady flow conditions. However, these two methods cannot be used in a moving-bed condition because both methods rely on accurate, unbiased bottom tracking. Since streamflow measurements are usually conducted in steady flow conditions and moving-bed often exists in rivers, the proposed trial-and-error method might be more applicable than the ‘course method’ or ‘multi-epoch method’.

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References

[6] H. Huang, Uncertainty model for in situ quality control of stationary ADCP open