The Size Resolution of the LISST Series of Instruments

SEQUOIA SCIENTIFIC, INC. ◆ *APPLICATION NOTE L008*

The LISST instruments observe the multi-angle scattering of light from an ensemble of particles in water. This multi-parameter measurement is used to solve (*invert*) a matrix equation that produces the size distribution. For details, please see Application Note 1. The present Application Note explains the resolution of particle sizes achievable by this method.

The relation between the data consisting of observed scattering at the multiple angles, $\underline{E}$, and the *area distribution* $\underline{N}_A$ of particles is expressed as:

$$\underline{E} = \mathbf{K}\,\underline{N}_A$$

where $\underline{N}_A$, the *area distribution* is defined as $\int a^2 n(a)da$. Each element of the vector $\underline{N}_A$ represents the integral performed over a small log-spaced size subrange. This represents the summation of the cross-sectional areas of particles within the specific size-range. The kernel matrix K has as its columns, the angular scattering signature of the particles. That is, each column represents the angular scattering that is expected from a single size-class of particles, when $\underline{N}_A$ is set to 1 for that size-class. This is a computed quantity. The computation employs Mie theory for spheres. Each element of the kernel matrix is the double integration of scattering over angles covered by a single ring of the detector, and covering the size subrange included in a size class.

To obtain the size distribution, first, the area distribution is solved for, by inverting the above equation. Next, the *volume distribution*, $\underline{N}_v$ (also frequently termed C_n in the literature on sediment transport) is obtained by a term-by-term multiplication of the elements of the area distribution by the mean diameter $d_{m,i}$ in the corresponding size class, or

$$\underline{N}_{v,i} = N_{A,I}\, d_{m,I}$$

where we ignore constant multiplier including π.

In the LISST instruments, there are 32 ring detectors which in principle, can be used to solve for 32 unknowns. That is, there should be 32 size classes at which the concentration can be determined. The question addressed in this Note is: Is this really so? Does the measurement permit one to distinguish two particles in adjacent size-classes? More specifically, in the type B instruments (1.25-250 micron measurement range), where adjacent size classes differ by 18%, can two particles barely 18% apart in size be recognized as to be of different sizes? Unfortunately, the answer is rather complicated.

If the data are perfect, i.e. noise-free, then it is possible to resolve the 32 sizes. In such a case, a simple inverse of the matrix **K** can be carried out and the area distribution becomes simply:

$$\underline{N}_A = \mathbf{K}^{-1}\underline{E}.$$

This is, however, hardly ever (in fact, *never*) done. The reason is that the matrix inverse $\mathbf{K}^{-1}$ usually amplifies any noise in the measurement and distorts the resulting estimate of size distribution. The property of the matrix that affects the severity of this distortion is called the *condition number* of the matrix. The interested (and mathematically inclined) reader is referred to two landmark papers that explain the details of the theory (Twomey, 1977 and Hirleman, 1987). For those not conversant in linear algebra, an explanation follows.

The concept that 32 unknowns can be solved for, given the 32 angles at which scattering is measured, is based on solving algebraic equations. Of course, they can be solved exactly. However, when the left-hand side of the equation is contaminated by measurement noise, one 'loses information'. Noise can make two equations identical, within the measurement error. This is equivalent to the loss of an equation, or loss of information. As a consequence, in practice, despite the availability of 32 measurements, only about 10-12 sizes may be resolved across the 200:1 size range observable by this type of instruments. This conclusion applies regardless of the inversion algorithm employed. Only when additional information is available, such as knowing the distribution to be narrow, or that it has a certain shape, can more resolution be obtained.

To show the performance of the LISST series, we display the result of laboratory tests with spheres. In a series of steps, smaller particles were added to filtered water. The particles were added in the following sequence: 165, 41, 17, 4.5 and 2.5 microns. Each set of particles was added in just sufficient quantity to make the resulting peak in the size distribution display of roughly the same height as the earlier peaks. The display below confirms the ability to resolve these sizes completely.

The sizes chosen for figure 1 are at typically at least a factor of 2 apart, not less. This has only to do with the availability of such particles. Nonetheless, the clear identification of the various size peaks, without the benefit of averaging the data, illustrates the basic capability of the instruments.

Other algorithms for special cases can be developed by the user or by request, at Sequoia Scientific, Inc. In case of interest, please contact the factory via phone or e-mail at info@sequoiasci.com.

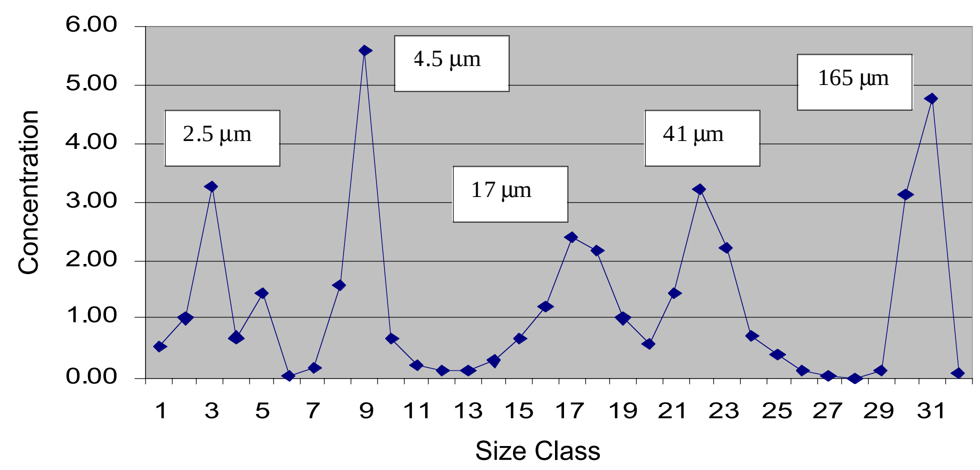

***Figure 1:** Particles of 5 distinct sizes were added to clean water. The results of inversion are displayed. For this data, an average of 100 scans of the data was inverted.*

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