Mackenzie

$$c(D,S,T) = 1448.96 + 4.591T - 5.304 \times 10^{-2}T^2 + 2.374 \times 10^{-4}T^3 + 1.340\,(S-35) + 1.630 \times 10^{-2}D + 1.675 \times 10^{-7}D^2 - 1.025 \times 10^{-2}T(S - 35) - 7.139 \times 10^{-13}TD^3$$

T = temperature in degrees Celsius
S = salinity in parts per thousand
D = depth in metres

Range of validity: temperature 2 to 30 °C, salinity 25 to 40 parts per thousand and depth 0 to 8000 m

The above equation for the speed of sound in sea-water as a function of temperature, salinity and depth is given by Mackenzie (1981).

Coppens

$$c(D,S,t) = c(0,S,t) + (16.23 + 0.253t)D + (0.213-0.1t)D^2 + [0.016 + 0.0002(S-35)](S - 35)tD$$

$$c(0,S,t) = 1449.05 + 45.7t - 5.21t^2 + 0.23t^3 + (1.333 - 0.126t + 0.009t^2)(S - 35)$$

t = T/10 where T = temperature in degrees Celsius
S = salinity in parts per thousand
D = depth in metres

Range of validity: temperature 0 to 35 °C, salinity 0 to 45 parts per thousand and depth 0 to 4000 m

The above equation for the speed of sound in sea-water as a function of temperature, salinity and depth is given by Coppens (1981).

UNESCO Equation: Chen and Millero

The international standard algorithm, often known as the UNESCO algorithm, is due to Chen and Millero (1977), and has a more complicated form than the simple equations above, but uses pressure as a variable rather than depth. For the original UNESCO paper see Fofonoff and Millard (1983). Wong and Zhu (1995) recalculated the coefficients in this algorithm following the adoption of the International Temperature Scale of 1990 and their form of the UNESCO equation is:

$$c(S,T,P) = Cw(T,P) + A(T,P)S + B(T,P)S^{3/2} + D(T,P)S^2$$

$$\begin{aligned} Cw(T,P) = {} & (C_{00} + C_{01}T + C_{02}T^2 + C_{03}T^3 + C_{04}T^4 + C_{05}T^5) + \\ & (C_{10} + C_{11}T + C_{12}T^2 + C_{13}T^3 + C_{14}T^4)P + \\ & (C_{20} + C_{21}T + C_{22}T^2 + C_{23}T^3 + C_{24}T^4)P^2 + \\ & (C_{30} + C_{31}T + C_{32}T^2)P^3 \end{aligned}$$

$$\begin{aligned} A(T,P) = {} & (A_{00} + A_{01}T + A_{02}T^2 + A_{03}T^3 + A_{04}T^4) + \\ & (A_{10} + A_{11}T + A_{12}T^2 + A_{13}T^3 + A_{14}T^4)P + \\ & (A_{20} + A_{21}T + A_{22}T^2 + A_{23}T^3)P^2 + \\ & (A_{30} + A_{31}T + A_{32}T^2)P^3 \end{aligned}$$

$$B(T,P) = B_{00} + B_{01}T + (B_{10} + B_{11}T)P$$

$$D(T,P) = D_{00} + D_{10}P$$

T = temperature in degrees Celsius
S = salinity in Practical Salinity Units (parts per thousand)
P = pressure in bar

Range of validity: temperature 0 to 40 °C, salinity 0 to 40 parts per thousand, pressure 0 to 1000 bar (Wong and Zhu, 1995).

Table of Coefficients

Coefficients	Numerical values	Coefficients	Numerical values
C_{00}	1402.388	A_{02}	7.166E-5
C_{01}	5.03830	A_{03}	2.008E-6
C_{02}	-5.81090E-2	A_{04}	-3.21E-8
C_{03}	3.3432E-4	A_{10}	9.4742E-5
C_{04}	-1.47797E-6	A_{11}	-1.2583E-5
C_{05}	3.1419E-9	A_{12}	-6.4928E-8
C_{10}	0.153563	A_{13}	1.0515E-8
C_{11}	6.8999E-4	A_{14}	-2.0142E-10
C_{12}	-8.1829E-6	A_{20}	-3.9064E-7
C_{13}	1.3632E-7	A_{21}	9.1061E-9
C_{14}	-6.1260E-10	A_{22}	-1.6009E-10
C_{20}	3.1260E-5	A_{23}	7.994E-12

Coefficients	Numerical values	Coefficients	Numerical values
C_{21}	-1.7111E-6	A_{30}	1.100E-10
C_{22}	2.5986E-8	A_{31}	6.651E-12
C_{23}	-2.5353E-10	A_{32}	-3.391E-13
C_{24}	1.0415E-12	B_{00}	-1.922E-2
C_{30}	-9.7729E-9	B_{01}	-4.42E-5
C_{31}	3.8513E-10	B_{10}	7.3637E-5
C_{32}	-2.3654E-12	B_{11}	1.7950E-7
A_{00}	1.389	D_{00}	1.727E-3
A_{01}	-1.262E-2	D_{10}	-7.9836E-6

Del Grosso's equation

An alternative equation to the UNESCO algorithm, which has a more restricted range of validity, but which is preferred by some authors, is the Del Grosso equation (1974). Wong and Zhu (1995) also reformulated this equation for the new 1990 International Temperature Scale and their version is:

$$c(S,T,P) = C_{000} + \Delta C_T + \Delta C_S + \Delta C_P + \Delta C_{STP}$$

$$\Delta C_T(T) = C_{T1}T + C_{T2}T^2 + C_{T3}T^3$$

$$\Delta C_S(S) = C_{S1}S + C_{S2}S^2$$

$$\Delta C_P(P) = C_{P1}P + C_{P2}P^2 + C_{P3}P^3$$

$$\Delta C_{STP}(S,T,P) = C_{TP}TP + C_{T3P}T^3P + C_{TP2}TP^2 + C_{T2P2}T^2P^2 + C_{TP3}TP^3 + C_{ST}ST + C_{ST2}ST^2 + C_{STP}STP + C_{S2TP}S^2TP + C_{S2P2}S^2P^2$$

T = temperature in degrees Celsius
S = salinity in Practical Salinity Units
P = pressure in kg/cm^2

Range of validity: temperature 0 to 30 °C, salinity 30 to 40 parts per thousand, pressure 0 to 1000 kg/cm^2, where100 kPa=1.019716 kg/cm^2. Wong and Zhu (1995)

Table of Coefficients

Coefficients	Numerical values
C_{000}	1402.392
C_{T1}	0.5012285E1
C_{T2}	-0.551184E-1
C_{T3}	0.221649E-3
C_{S1}	0.1329530E1
C_{S2}	0.1288598E-3
C_{P1}	0.1560592
C_{P2}	0.2449993E-4
C_{P3}	-0.8833959E-8
C_{ST}	-0.1275936E-1
C_{TP}	0.6353509E-2
C_{T2P2}	0.2656174E-7
C_{TP2}	-0.1593895E-5
C_{TP3}	0.5222483E-9
C_{T3P}	-0.4383615E-6
C_{S2P2}	-0.1616745E-8
C_{ST2}	0.9688441E-4
C_{S2TP}	0.4857614E-5
C_{STP}	-0.3406824E-3

Both the UNESCO equation and Del Grosso's equation use pressure rather than depth as a variable. Useful guidance and suitable equations for converting pressure into depth and depth into pressure can be found in Leroy and Parthiot (1998) . The key equations here are:

Conversion of pressure into depth

$$Z_S(P, \Phi) = \frac{9.72659 \times 10^2 P - 2.512 \times 10^{-1} P^2 + 2.279 \times 10^{-4} P^3 - 1.82 \times 10^{-7} P^4}{g(\Phi) + 1.092 \times 10^{-4} P}$$

Where g(Φ), the international formula for gravity, is given by:

$$g(\Phi) = 9.780318 (1 + 5.2788 \times 10^{-3} \sin^2\Phi + 2.36 \times 10^{-5} \sin^4\Phi)$$

Z = depth in metres
P = pressure in MPa
Φ = latitude

The above equation is true for the oceanographers' standard ocean, defined as an ideal medium with a temperature of 0 °C and salinity of 35 parts per thousand.

Leroy and Parthiot (1998) give a table of corrections which are needed when the standard formula is applied to specific oceans and seas. The above equation and interactive version do not apply any corrections.

Conversion of depth into pressure

$$h(Z\Phi) \quad h(Z,45) \times k(Z,\Phi)$$

$$h(Z,45) = 1.00818 \times 10^{-2} Z + 2.465 \times 10^{-8} Z^2 - 1.25 \times 10^{-13} Z^3 + 2.8 \times 10^{-19} Z^4$$

$$k(Z,\Phi) \quad (g(\Phi) - 2 \times 10^{-5} Z)/(9.80612 - 2 \times 10^{-5} Z)$$

$$g(\Phi) = 9.7803(1 + 5.3 \times 10^{-3} \sin^2\Phi)$$

Z = depth in metres
h = pressure in MPa
Φ = latitude

The above equation is true for the oceanographers' standard ocean, defined as an ideal medium with a temperature of 0 °C and salinity of 35 parts per thousand.

Leroy and Parthiot (1998) give a table of corrections which are needed when the standard formula is applied to specific oceans and seas. The above equation and interactive version do not apply any corrections.

Uncertainties

Although the UNESCO algorithm is the international standard algorithm, there is much debate in the scientific literature about the accuracy and range of applicability of this equation and of Del Grosso's equation. Some researchers prefer Del Grosso's equation, especially for calculations within its own domain of validity. It is important to recognise that the equations presented here are derived from fitting to experimental data from several different experiments and each has an associated uncertainty in its prediction of sound speed. The choice of equation may depend on the accuracy and precision which is acceptable for the particular application in which it is being employed. For further discussion on this topic please refer to Dushaw et al (1993), Meinen and Watts (1997), Millero and Xu Li (1994), Speisberger and Metzger (1991a, 1991b) and Speisberger (1993).

The Hydrographic Society (Pike and Beiboer, 1993) has a good summary of the main algorithms for sound speed in the ocean. This sets out more detailed advice and information than can be provided on this web-site, including information of the domains of validity of the main equations and on depth to pressure conversions.

Any comments or suggestions about further speed of sound equations?

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